Fatigue life estimate of landing Gear's leg using modal analysis

Essam A. Al-Bahkali^{1,*}, Hisham Elkenani², and Mhamed Souli³

¹Department of Mechanical Engineering King Saud University P.O. Box 800, Riyadh 11421, Saudi Arabia. ebahkali@ksu.edu.sa ²Department of Mechanical Engineering King Saud University P.O. Box 800, Riyadh 11421, Saudi Arabia. hkenani@ksu.edu.sa ³Laboratoire de Mécanique, de Lille, UMR CNRS 8107, Villeneuve d'Ascq, France, mhamed.souli@univ-lille1.fr

ABSTRACT

The present work concerns solving Noise, Vibration and Harshness (NVH) and fatigue based on Power Spectrum Density (PSD) analysis of a landing gear's leg for an Un-Manned Aerial Vehicle (UAV). This analysis includes random vibration and high-cycle fatigue analysis in a random vibration environment.

In this analysis, the cumulative damage ratio is computed using material S-N (Stress-Number of cycles) fatigue curve. Dirlik method is used for the analysis of lifetime as it is proven to provide accurate results for large number of applications, both in automotive and aerospace industry. It is also compared to other methods that have been developed in LS-DYNA® as well. The input acceleration PSD data are provided through measurements.

The obtained analysis results shows that although the landing gear design is safe according to dynamic and static load, its service life is about 3037 hours due to random vibration effect.

Keywords: Landing gear, Power spectrum density, Fatigue, Vibration

1. INTRODUCTION

Most real fatigue loadings are random processes in respect of frequency and stress amplitude. Estimating fatigue damage with Power Spectrum Density (PSD) was first proposed by Rice [1] in 1954. The stress power spectra density (PSD) represents the frequency domain approach input into the fatigue. This is a scalar function that describes how the power of the time signal is distributed among frequencies [2]. Mathematically this function can be obtained by using a Fourier transform of the stress time history's autocorrelation function, and its area represents the signal's standard deviation. It is clear that PSD is the most complete and concise representation of a random process [3]. Finite element analysis is very efficient in taking the PSD of applied loads and determining the PSD of the resulting stresses at various points in the structure. A method of taking the PSD of stress and calculating fatigue lives therefore has attractions. Much of the early work on fatigue analysis from PSD's was carried out by NASA in order to determine the fatigue damage caused by vibration and buffeting of space vehicles.

* .

^{*}Corresponding author: Email: ebahkali@ksu.edu.sa

2. RANDOM RESPONS ANALYSIS

Random response linear dynamic analysis is used to predict the response of a structure subjected to a nondeterministic continuous excitation that is expressed in a statistical sense by a cross-spectral density (CSD) matrix. The random response procedure uses the set of eigenmodes extracted in a previous eigenfrequency step to calculate the corresponding power spectral densities (PSD) of response variables (stresses, strains, displacements, etc.) and, hence—if required—the variance and root mean square values of these same variables. This chapter provides brief definitions and explanations of the terms used in this type of analysis based in the book by Clogh and Penzien [4].

Examples of random response analysis are the study of the response of an airplane to turbulence; the response of a car to road surface imperfections; the response of a structure to noise, such as the "jet noise" emitted by a jet engine; and the response of a building to an earthquake.

Since the loading is nondeterministic, it can be characterized only in a statistical sense. We need some assumptions to make this characterization possible. Although the excitation varies in time, in some sense it must be stationary—its statistical properties must not vary with time. Thus, if x(t) is the variable being considered (such as the height of the road surface in the case of a car driving down a rough road), then any statistical function of x, f(x), must have the same value regardless of what time origin we use to compute f:

$$f(x(t)) = f(x(t+\tau))$$
 for any τ (1)

We also need the excitation to be ergodic. This term means that, if we take several samples of the excitation, the time average of each sample is the same. These restrictions ensure that the excitation is, statistically, constant.

The following subchapters explain some concepts used in random response analysis.

2.1. STATISTICAL MEASURES

The mean value of a random variable x(t) is:

$$E(x) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt.$$
 (2)

Since the dynamic response is computed about a static equilibrium configuration, the mean value of any dynamic input or response variable will always be zero:

$$E(x) = 0. (3)$$

The variance of a random variable measures the average square difference between the point value of the variable and its mean:

$$\sigma_r^2(x) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (x(t) - E(x))^2 dt.$$
 (4)

Since E(x) = 0 for our applications, the variance is the same as the mean square value:

$$E\left(x^{2}\right) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^{2}\left(t\right) dt. \tag{5}$$

The units of variance are (amplitude)², and are expressed as root mean square (RMS) values:

$$\sigma_r(x) = \sqrt{\sigma_r^2} \,. \tag{6}$$

2.2. CORRELATION

Correlation measures the similarity between two variables. Thus, the cross-correlation between two random functions is the integration of the product of the two variables, with one of them shifted in time by some fixed value to allow for the possibility that they are similar but shifted in time.

The cross-correlation function for the variables is defined as:

$$R_{x_1 x_2}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x_1(t) x_2(t+\tau) dt.$$
 (7)

Since the mean value of any variable is zero, on average each variable has equal positive and negative content. If the variables are quite similar, their cross-correlation for some values of (τ) will be large; if they are not similar, the product x_1x_2 will sometimes be negative and sometimes positive so that the integral over all time will provide a much smaller value, regardless of the choice of τ . A simple result is:

$$R_{x_1x_2}(\tau) = R_{x_2x_1}(-\tau). \tag{8}$$

The autocorrelation is the cross-correlation of a variable with itself. Then the autocorrelation of a variable x(t) is defined as:

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) x(t+\tau) dt.$$
 (9)

 $R(\tau) \rightarrow \sigma_r^2$, the autocorrelation equals the variance (the mean square Clearly, as $\tau \to o$, value).

The autocorrelation, $R(\tau)$ tells us about the nature of the random variable. If $R(\tau)$ drops off rapidly as the time shift τ moves away from $\tau = 0$, the variable has a broad frequency content; if it drops off more slowly and exhibits a cosine profile, the variable has a narrow frequency content centered around the frequency corresponding to the periodicity of $R(\tau)$.

This concept is extended to detect the frequency content of a random variable by crosscorrelating the variable with a sine wave: sweeping the wave over a range of frequencies and examining the cross-correlation tells us whether the random variable is dominated by oscillation at particular frequencies. We begin to see that the nature of stationary, ergodic random processes is best understood by examining them in the frequency domain.

We can write x(t), which contains many discrete frequencies in terms of a Fourier series

expanded in N steps of a fundamental frequency
$$\omega_0$$
:
$$x(t) = \sum_{n=1}^{N} \left[a_n \cos(in \, \omega_0 t) + b_n \sin(in \, \omega_0 t) \right] \tag{10}$$

Keeping in mind that we will be interested only in the real part, the variance of x(t) can be expressed as:

$$\sigma_r^2(x) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-N}^{N} A_n^* \exp(-in\,\omega_0 t) A_n \exp\left(in\,\omega_0 t\right) dt. \tag{11}$$

Where A_n is the complex amplitude of the nth term, A^* is the complex conjugate of A_n . Using the orthogonality of Fourier terms. Continuing: $\sigma_r^2(x) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=1}^{N} A_n^* A_n dt$

$$\sigma_r^2(x) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{n=-N}^{N} A_n^* A_n dt$$

$$= \sum_{n=-N}^{N} A_n^* A_n = \sum_{n=-N}^{N} |A_n|^2 = \sum_{n=-N}^{N} \sigma_r^2(x_n)$$
(12)

where x_n is the *n*th component of the Fourier series.

The contribution to the variance of x, $\sigma_r^2(x)$, at the frequency $f_n = n\omega_0/(2\pi)$, per unit frequency, is thus:

$$S_x(f_n) = \frac{2\pi}{\omega_0} A_n A_n^*. \tag{13}$$

since we are stepping up the frequency range in steps of $\Delta f = \omega_0 / (2\pi)$. The variance can, therefore, be written as:

$$\sigma_r^2(x) = \sum_{n=-N}^N S_x(f_n) \Delta f \tag{14}$$

As we examine x as a function of frequency, $S_x(f_n)$ tells us the amount of "power" (in the sense of mean square value) contained in x, per unit frequency, at the frequency f_n . As we consider smaller and smaller intervals, $\Delta f \rightarrow 0$, S_x is the power spectral density (PSD) of the variable x:

$$\sigma_r^2(x) = \int_{-\infty}^{\infty} S_x(f) df. \tag{15}$$

2.3. RANDOM RESPONSE ANALYSIS

For a system excited by some random loads or prescribed base motions, which are characterized in the frequency domain by a matrix of cross-spectral density functions, S_{NM} (f), we can think of N and M as two of the degrees of freedom of the finite element model that are exposed to the random loads or prescribed base motions. In typical applications the range of frequencies will be limited to those to which we know the structure will respond—we do not need to consider frequencies that are higher than the modes in which we expect the structure to respond.

The values of $S_{NM}(f)$ might be provided by Fourier transformation of the cross-correlation of time records or by the Fourier transformation of the autocorrelation of a single time record, together with known geometric data. The system will respond to this excitation. We are usually interested in looking at the power spectral densities of the usual response variables—stress, displacement, etc. The PSD history of any particular variable will tell us the frequencies at which the system is most excited by the random loading.

An overall picture is provided by looking at the variance (the mean square value) of any variable; the RMS value is provided for this purpose. The RMS value is used instead of variance because it has the same units as the variable itself. It is computed by integrating the single-sided power spectral density of the variable, S_x over the frequency range.

The transformation of the problem into the frequency domain inherently assumes that the system under study is responding linearly.

2.4. THE FREQUENCY RESPONSE FUNCTION (FRF)

Random response is studied in the frequency domain. Therefore, we need the transformation from load to response as a function of frequency. Since the random response is treated as the integration of a series of sinusoidal vibrations. The discrete (finite element) linear dynamic system has the equilibrium equation:

$$\delta u^{N} \left(M^{NM} \ddot{u}^{M} + C^{NM} \dot{u}^{M} + K^{NM} u^{M} \right) = \delta u^{N} F^{N}. \tag{16}$$

where M^{NM} is the mass matrix, C^{NM} is the damping matrix, K^{NM} is the stiffness matrix, F^{N} are the external loads, u^{N} is the value of degree of freedom N of the finite element model, and δu^{N} is an arbitrary virtual variation.

The problem is projected onto the eigenmodes of the system, which are first extracted from the undamped system. The eigenmodes are orthogonal across the mass and stiffness matrices.

2.5. VON MISES STRESS COMPUTATION

This is a quite critical issue, which has been largely investigated in the literature. Noting that the von Mises stress is a quadratic stress function, therefore it is a non-Gaussian stress with positive mean value, even if its stress components are zero-mean Gaussian random stresses.

The first one and theoretical proper definition of the Mises stress in frequency domain was proposed by Preumont and Piefort [5]. An interesting paper about including the phase shift into von Mises PSD were written by Bonte [6].

The computation of PSD and RMS of von Mises stresses in this chapter is based on work by Segalman, et al. [7]. Under this approach the PSD of von Mises stress at a node a is given by:

$$S_{mises}^{a}(f) = \sum_{\beta=1}^{m} \sum_{\alpha=1}^{m} S_{\alpha\beta}(f) T_{\alpha\beta}^{\alpha}$$
(17)

where m is the number of modes, $S_{\alpha\beta}(f)$ are the elements of the PSD matrix of Ψ generalized displacements:

$$T_{\alpha\beta}^{\alpha} = \left[\downarrow_{\beta}^{\alpha} \right]^{T} [A] \left[\downarrow_{\beta}^{\alpha} \right]. \tag{18}$$

 $\left[\downarrow_{\beta}^{\alpha} \right]$ are the modal stress components of the α th mode at node a, and the constant matrix A is given by:

$$\begin{pmatrix}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 \\
\frac{-1}{2} & 1 & \frac{-1}{2} & 0 & 0 & 0 \\
\frac{-1}{2} & \frac{-1}{2} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 3
\end{pmatrix}.$$
(19)

Similarly, RMS of von Mises stress at a node a is computed as:

$$RMISES^{a}\left(f\right) = \sqrt{\sum_{\beta=1}^{m} \sum_{\alpha=1}^{m} V_{\alpha\beta}\left(f\right) T_{\alpha\beta}^{a}}.$$
 (20)

where $V_{\alpha\beta}(f)$ are the elements of the variance matrix of generalized displacements. Figure 1 summarize the spectral analysis of the stationary random response.

3. FREQUENCY DOMAIN APPROACHES OF LIFE ESTIMATION

Fatigue analysis can be conducted in both time and frequency domain. In time domain, rainflow counting algorithm is usually used to calculate the number of cycles at each stress and strain levels based on the time history. Due to random load in many cases, a description in

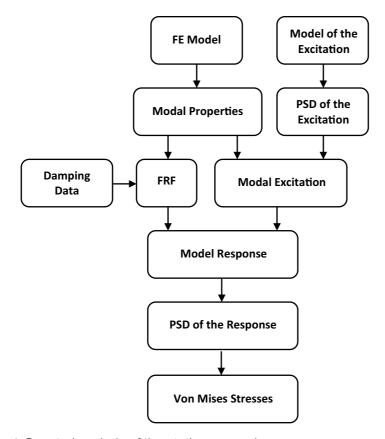


Figure 1: Spectral analysis of the stationary random response

frequency domain is more efficient and hence using a statistical method is the most appropriate approach.

There are seven frequency domain fatigue analysis methods have been implemented in LS-DYNA. Figure 2 illustrates these method and their common applications.

All of the shown methods are based on Palmgren-Miner's rule of cumulative damage ratio:

$$E[D] = \sum_{i} \frac{n_i}{N_i} \tag{21}$$

Where E[D] is the expected damage ratio, n_i is the number of cycles at stress level S_i , and N_i is the number of cycles for failure at stress level S_i , given by material's S-N curve.

To get n_i from the PSD of the random stress response and further compute E[D], a variety of statistical methods have been used to predict Probability Density Function (PDF) and rainflow count, namely:

Dirlik, Steinberg, Narrow Band, Wirsching, Chaudhury and Dover, Tunna and Hancock method.

3.1. DIRLIK METHOD

The Dirlik method is a mathematical tool which used to performing fatigue analysis. To use this method, signals to be analysed must first be transferred from the time domain to

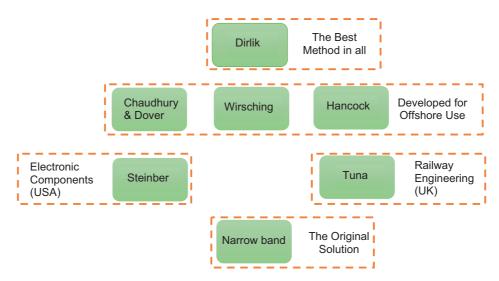


Figure 2: Frequency domain methods and their common applications

the frequency domain where they are presented in the form of a power spectral density (PSD) function. Stress analysis in the time domain is ideally suited to signals representing periodic or continuous stress loading but for random stress loading data, prohibitively large time records are often required for an accurate analysis. There are distinct advantages in performing stress analysis in the frequency domain rather than the time domain where a random stress loading signal forms the basis for analysis. The main advantage is much less time intensive calculations although this comes at a cost of slight loss in accuracy.

Any fatigue analysis process begins with the response of the structure or component. This is normally expressed as a time history of stress or strain. Stress cycles in the time history result in fatigue and the most important aspects of these cycles are the stress amplitude ranges and the mean stress values. These values are normally analysed using Rainflow Cycle Counting. The time history data can be converted to data in the frequency domain by using a variation of the Fourier Transform. In both formats, the y-axis displays the signal amplitude but in the frequency domain, the x-axis represents the signal frequency as opposed to time. The Fourier Transform effectively breaks the signal down into discrete sinusoidal waves. These waves vary in frequency, phase and amplitude and form the original time signal again when combined using an inverse Fourier Transform.

The Dirlik method [8][9], devised in 1985, approximates the cycle- amplitude distribution by using a combination of one exponential and two Rayleigh probability densities. It is based on numerical simulations of the time histories for two different groups of spectra. This method has long been considered to be one of the best methods and has already been subject to modifications, e.g., for the inclusion of the temperature effect, by Zalaznik and Nagode [10].

The Dirlik method consists of a series of calculations which are based on four moments of area of the PSD function and was taken from Halfpenny [11]. These moments of area are m_0 , m_1 , m_2 and m_4 . The nth moment of area is calculated as:

$$m_n = \int f^n \cdot G(f) df \tag{22}$$

where G(f) is the PSD function and f is the frequency in Hertz

The rainflow cycle amplitude probability density, p_a estimate is given by:

$$p_a(S) = \frac{1}{\sqrt{m_0}} \left[\frac{G_1}{Q} e^{\frac{-Z}{Q}} + \frac{G_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + G_3 Z e^{\frac{-Z^2}{2}} \right]$$
(23)

where Z is the normalized amplitude:

$$Z = \frac{S}{\sqrt{m_0}} \tag{24}$$

For a fatigue analysis the moments up to m_4 are normally used. The even moments represent the variance σ_x^2 of the random process X and its derivatives:

$$\sigma_X^2 = m_0 \qquad \sigma_{\dot{X}}^2 = m_2 \tag{25}$$

and the parameters G1 to G3, R and Q are defined as:

$$G_{1} = \frac{2(x_{m} - \alpha_{2}^{2})}{1 + \alpha_{2}^{2}} \qquad G_{2} = \frac{1 - \alpha_{2} - G_{1} + G_{1}^{2}}{1 - R}$$

$$G_{3} = 1 - G_{1} - G_{2} \qquad R = \frac{\alpha_{2} - x_{m} - G_{1}^{2}}{1 - \alpha_{2} - G_{1} + G_{1}^{2}}$$

$$Q = \frac{1,25(\alpha_{2} - G_{3} - G_{2}R)}{G_{1}} \qquad (26)$$

where x_m is the mean frequency, as defined by the author of the method [8]:

$$x_m = \frac{m_1}{m_0} \left(\frac{m_2}{m_4} \right)^{\frac{1}{2}} \tag{27}$$

where α_i is used for spectral width estimation and it has the general form:

$$\alpha_i = \frac{m_i}{\sqrt{m_0 m_{2i}}} \tag{28}$$

4. FREQUENCY DOMAIN RANDOM VIBRATION ANALYSIS

The random vibration capability of LS-DYNA® originated from Boeing's in-house vibroacoustic code N-FEARA [12]. This feature computes the dynamic response of structures exposed to vibration or structural-acoustic coupling based on a known source. Various excitations and acoustic environments can be considered, including base excitations, correlated or non-correlated acoustic waves such as plane wave, progressive wave, reverberant wave, turbulent boundary layer, etc. [13]

A keyword has been introduced in LS_DYNA® to perform random vibration analysis. Through the keyword, user provides information about the location, direction, range of frequencies for the random excitation and response area can be given as node, set of nodes, set of segments, or part. The direction of load can be in any of the x, y, z directions or given as a vector. Load curve IDs for the PSD loads in random computation are also specified under the keyword [14].

The feature of random vibration fatigue is implemented as an option of the keyword, as it is a natural extension of the random analysis procedure. The method for performing fatigue analysis is defined as well as the parts or elements where the fatigue analysis is needed. The material's S-N fatigue curve and some other options like the exposure time are also defined.

Modal analysis is the first step for running the analysis and then the implicit solution started.

Ringeval A. and Huang Y. [14] give in their paper several examples to demonstrate the effectiveness of the random vibration fatigue analysis feature with LS-DYNA®. One of those examples is a simple cantilever aluminium beam [15] subjected to base accelerations. The numerical values are compared with the experimental results and different fatigue failure theories are used to predict fatigue life. That example is considered as a benchmark for the verification of the results obtained with LS-DYNA® as the authors compare their results with those found in the literature on the same subject. Also, the same problem has been simulated with ANSYS® and RADIOSS® BULK and the maximum root mean square (RMS) stress computed by LS-DYNA® is in good agreement with the results obtained by the other commercial software.

5. LANDING GEAR DESIGN

The landing gear is a structure that supports an aircraft on ground and allows it to taxi, take off, and land [15]. In fact, landing gear design tends to have several interferences with the aircraft structural design. Nowadays the weight of landing gear has become an important factor. Efforts are being made to reduce the weight of the aircraft and consequently increase the payload. Design of landing gears takes the effect of static and dynamic loads as well as the impact loading. This paper presents analysis for prediction of the service life for a light landing gear with a thickness optimized to only 8 millimeters. This design optimization where carried by Albahkali E. and Alqhtani M. [17]. Dimensions of the landing gear are shown in Figure 1.

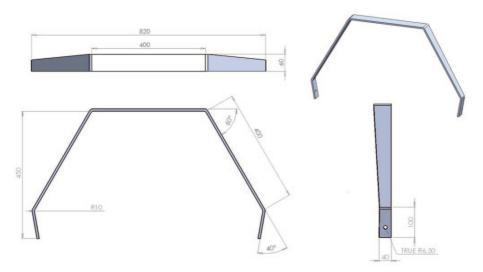


Figure 3: Geometry details of the landing gear (all dimensions are in millimeter)

As shown in Figure 4, the landing gear is clamped to the fuselage in its middle horizontal part. The two wheels attached to its ends are weighing 0.651 kg each.

Figure 5 shows Power Spectral Density (PSD) outputs in g^2/Hz for the landing gear vibration with change in frequency. It indicates random vibration loads on the landing gear. The modal frequencies required to calculate the resultant effect of modal spectrum vibration are extracted up to the spectrum frequency.



Figure 4: Landing gear

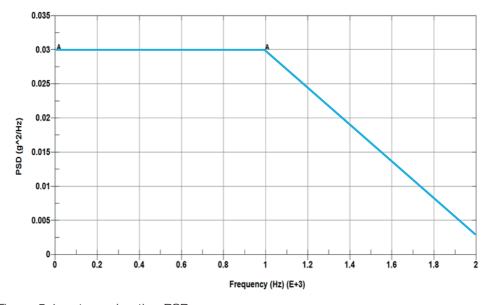


Figure 5: Input acceleration PSD

The landing gear is made from AA7076-T6 aluminum alloy with density ρ = 2700 kg/m3, Young's modulus E = 70 G Pa and Poisson's ratio ν = 0.33. The S-N fatigue curve, Figure 4, is selected according to the European Standard Eurocode 9 [18] with a reference fatigue strength $\Delta \sigma$ = 71 MPa at 2.106 cycles and a single inverse constant slope m = 7.0.

6. ANALYSIS STEPS

The model is meshed with 9073 nodes and 8576 shell elements (fully integrated linear DK quadrilateral and triangular shell). The two wheels are represented by 28 mass elements with 0.0465 kg each. They distributed uniformly at the circumference of the two holes. The upper face is subjected to base Acceleration Spectral Density for the range of frequency 0-2000 Hz. A constant damping ratio 0.035 is adopted. The landing gear is exposed to the random vibration load for 45 minutes (2700 seconds) which is equal to the flight time.

The first 10 natural modes are required for the eigenvalue analysis. Lobatto's integration rule with three integration points through the thickness of the landing gear is applied to get the stress on the shell surface. The shear factor which scales the transverse shear stress, is suggested to have a value equal to 5/6. For shell element with implicit solution the invariant node numbering is taken as 2. An infinite fatigue life for stress lower than the lowest stress on S-N curve (40.6 MPa) is assumed.

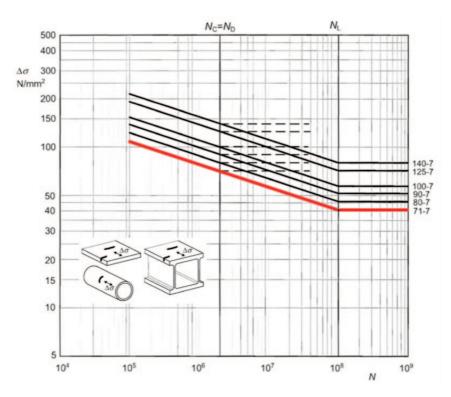


Figure 6: S-N fatique curve used for the landing gear material

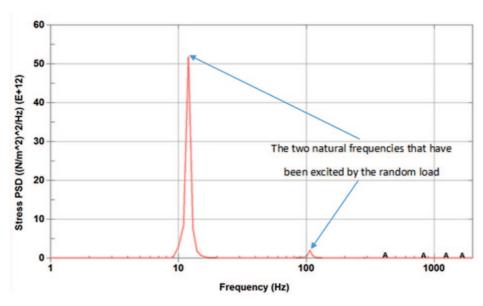


Figure 7: Stress PSD at critical point

Table 1 Fatigue life obtained by the numerical predictions methods used in LS-DYNA®.

Method used	Fatigue life (Hrs.)	Cumulative fatigue ratio
Steinberg	994.1	0.000754
Dirlik	3037.6	0.000246
Narrow Band	386.7	0.001939
Wirsching	1389.9	0.000539
Chaudhury and Dover	1035.1	0.000724
Tunna	67021.7	0.000011
Hancock	1040.0	0.000721

7. RESULTS

The response stress PSD measured at the critical point shows that two natural frequencies are excited by the input loading in the range 10-2000 Hz (Figure 5), the first natural frequency (11.711 Hz) and the fifth natural frequency (105.77 Hz). When examine the model with higher input acceleration PSD load ($g^2/Hz = 0.1$), the eighth natural frequency (287.244 Hz) is excited also. Table 1 summarizes the fatigue life obtained by the numerical predictions methods.

As shown in Table 1, the results depend on the method used to interpret the RMS results. Steinberg, Chaudhury and Dover and Hancock methods give close results but conservative. Wirsching method gives less conservative results. Tunna's prediction is completely off. Narrow Band's prediction is too conservative comparing to the other methods. Figure 8 and Figure 9 show contour plot for the cumulative damage ratio and the RMS of Sx stress at the critical point using Dirlik method respectively.

Based on Dirlik method results, the service life for the landing gear will be equal to 3037.6 Hrs. or 4050 flight trips.

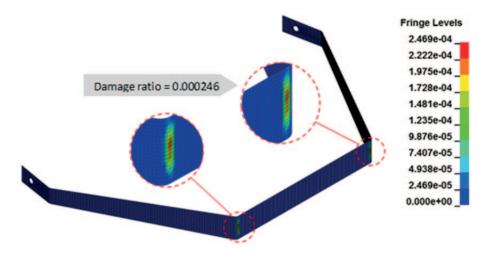


Figure 8: Cumulative damage ratio by Dirlik method

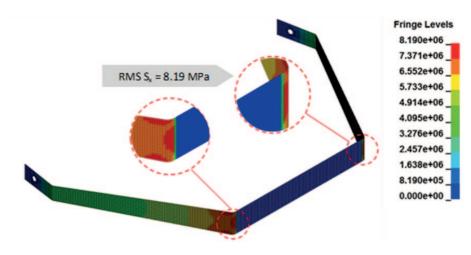


Figure 9: RMS of S_x stress at the critical point using Dirlik method

8. CONCLUSION

This paper presents solving frequency domain random vibration fatigue. The analysis provides cumulative damage ratio calculation and fatigue life prediction for a typical UAV landing gear subjected to random vibration excitations based on various theories. Dirlik method which has long been considered to be one of the best methods is demonstrated.

The conservatism associated with Narrow Band solution method is clear as it gives the lowest predicted service life. On other side, Steinberg method didn't give excessive non-conservative results as the applied random signal is a broad-band one.

In design of a structure in consideration of fatigue life, it is very important to evaluate not only a fatigue life under the constant loading but also a fatigue life under the service loading. Due to weight constrain, as in the case of the landing gear design, the margin of the factor of safety is limited. Therefore, it is essential to conduct fatigue analysis also to predict the service life. In addition, the frequency response approach can improve understanding of the system dynamic behaviors, in terms of frequency characteristics of both structures and loads and their couplings.

REFERENCES

- [1] Rice S., Mathematical analysis of random noise. Selected papers on noise and stochastic process, Dover, New York, 1954.
- [2] Bendat J., Probability functions for random responses: prediction of peaks, fatigue damage, and catastrophic failures, NASA report, 1964.
- [3] Braccesi C., Cianetti F., Lori G., Pioli D., Fatigue behaviour analysis of mechanical components subject to random bimodal stress process: frequency domain approach, *International Journal of Fatigue*, 2005,27, pp. 335–345.
- [4] Clough, R. W., and J. Penzien, Dynamics of Structures, McGraw-Hill, New York, 1975.
- [5] A. Preumont and V. Piefort, Predicting Random High-Cycle Fatigue Life With Finite Elements, *Journal of Vibration and Acoustics*, 116(2), pp. 245–248, Apr. 1994.
- [6] M. H. A. Bonte, A. de Boer, and R. Liebregts, Determining the von Mises stress power spectral density for frequency domain fatigue analysis including out-of-phase stress components, *Journal of Sound and Vibration*, 302(1–2), pp. 379–386, Apr.2007.
- [7] Segalman, D. J., C. W. G. Fulcher, G. M. Reese, and R. V. Field, Jr., An Efficient Method for Calculating RMS von Mises Stress in a Random Vibration Environment, Sandia Report, SAND98-0260, 1998.
- [8] Dirlik T., Application of computers in fatigue analysis, Ph.D. thesis. The University of Warwick, 1985.
- [9] Mršnik M., Slavic J., Boltezar M., Frequency-domain methods for a vibration-fatigue-life estimation Application to real data, *International Journal of Fatigue*, 2013, 47 (February), pp. 8–17.
- [10] Zalaznik A, Nagode M., Frequency based fatigue analysis and temperature effect, *Materials and Design*, 2011, 32(10), pp. 4794–802.
- [11] Halfpenny A., A frequency domain approach for fatigue life estimation from Finite Element Analysi, nCode® International Ltd, Sheffield UK.
- [12] LS-DYNA® Keyword User's Manual, Version 971, Livermore Software Technology Corporation, Livermore, California, 2012.
- [13] Huang Y., Souli M., Ashcraft C., Grimes R., Wang j., Rassaian M., Lee J., Development of frequency domain dynamic and acoustic capability in LS-DYNA®, *Proceedings of the 8th European LS-DYNA® Users Conference*, Strasbourg, May 2011.
- [14] Ringeval A., Huang Y., Random vibration fatigue analysis with LS-DYNA®, *Proceedings of the 12th International LS-DYNA® Users Conference*, Dearborn, Michigan, USA, June 3–5, 2012.
- [15] V. Nagulpalli, A. Gupta, S. Fan, Estimation of fatigue life of aluminum beams subjected to random vibration, Department of Mechanical Engineering, Northen Illinois University, DeKalb, IL.
- [16] Matta A., Kumar G., Kumar R., Design optimisation of landing gear's leg for an unmanned aerial vehicle, *International Journal of Engineering Research and Applications (IJERA)* ISSN: 2248-9622, 2012, 2(4), pp. 2069-2075.
- [17] Albahkali E., Alqhtani M., Design of light landing gear, Graduate Project, King Saud University, June, 2011.
- [18] Eurocode 9: Design of aluminium structures Part 1-3: Structures susceptible to fatigue, August 2011.