A mathematical theorem on the onset of Couple-Stress fluid permeated with suspended dust particles saturating a porous medium

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ABSTRACT
In this paper, the effect of suspended particles on thermal convection in Couple-Stress fluid saturating a porous medium is considered. By applying linear stability theory and normal mode analysis method, a mathematical theorem is derived which states that the viscoelastic thermal convection at marginal state, cannot manifest as stationary convection if the thermal Rayleigh number R, the medium permeability parameter P_l, the couple-stress parameter F and suspended particles parameter B, satisfy the inequality

$$R \le \frac{4\pi^4 F}{BP_l}.$$

Keywords: Couple-Stress fluid, Porous medium, Suspended particles, Thermal convection.

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1. INTRODUCTION

The problem of thermal convection in porous media has attracted considerable interest during the last few decades, because it has various applications in geophysics, food processing, soil sciences, ground water hydrology and nuclear reactors etc. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3]. Scanlon and Segel [4] have considered the effect of suspended particles on the onset of Be′nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

In all the above studies, the fluid is considered to be Newtonian. Although the problem of thermal convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. With the growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations on such fluids are desirable. One such type of fluid is couple-stress fluid. Stokes [5] proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as lubricant. When fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of human body and these joints have low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid.

According to the theory of Stokes [5], couple-stresses are found to appear in noticeable magnitude in fluids very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka [6] modeled synovial fluid as couple-stress fluid in human joints. Sharma and Sharma [7] have studied the couple-stress fluid heated from below in porous medium.

The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [8], Ingham and Pop [9] and Nield and Bejan [10]. Sharma and Rana [11] have studied thermal instability of a incompressible Walters' (model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium whereas Rana and Kumar [12] studied thermal instability of Rivlin-Ericksen elastico-viscous rotating fluid permitted with suspended particles and variable gravity field in porous medium. Recently, Kumar [13] studied stability of stratified couple-stress dusty fluid in the presence of magnetic field through porous medium whereas Rana and Sharma [14] studied hydromagnetic thermosolutal instability of compressible Walters' (model B′) rotating fluid permeated with suspended particles in porous medium.

The interest for investigations of non-Newtonian fluids is also motivated by a wide range of engineering applications which include ground pollutions by chemicals which are non-Newtonian like lubricants and polymers and in the treatment of sewage sludge in drying beds. Recently, polymers are used in agriculture, communications appliances and in bio medical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

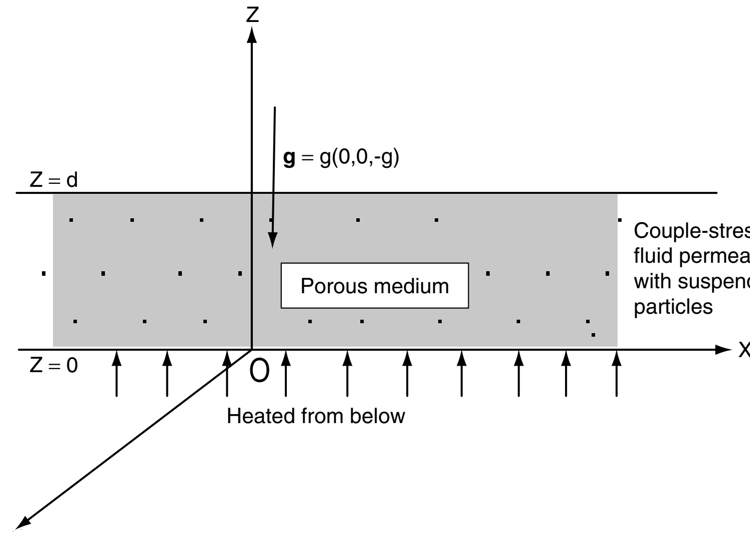
Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the effect of suspended particles on thermal instability of incompressible couple-stress elastico-viscous fluid in a porous medium.

2. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Here, we consider an infinite, horizontal, incompressible couple-stress viscoelastic fluid of depth d, bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity $\mathbf{g}(0, 0, -g)$. This layer is heated from below such that a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let ρ, υ, μ_c, p, ε, T, α and $v(0, 0, 0)$, denote respectively, the density, kinematic viscosity, couple-stress viscosity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass, temperature and equation of state for couple-stress fluid in a porous medium (Chandrasekhar [1], Sharma and Sharma [11], Kumar [13]) are



Schematic sketch of physical situation

$$\frac{1}{\varepsilon}\left[\frac{\partial v}{\partial t}+\frac{1}{\varepsilon}\left(\boldsymbol{v}\cdot\nabla\right)\boldsymbol{v}\right]=-\frac{1}{\rho_0}\nabla p+g\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\boldsymbol{v}-\frac{\mu_c}{\rho_0}\nabla^2\right)\boldsymbol{v}+\frac{K'N}{\rho_0\in}\left(\boldsymbol{v}_d-\boldsymbol{v}\right), \quad (1)$$

$$\nabla\cdot\boldsymbol{v}=0, \quad (2)$$

$$E\frac{\partial \mathrm{T}}{\partial t}+\left(\boldsymbol{v}\cdot\nabla\right)T+\frac{mNC_{pt}}{\rho_0 C_f}\left[\in\frac{\partial}{\partial t}+v_d\cdot\nabla\right]T=\kappa\nabla^2\mathrm{T}, \quad (3)$$

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right], \quad (4)$$

where the suffix zero refers to values at the reference level z = 0.

Here v_d ($\bar{x}$, t) and N ($\bar{x}$, t) denote the velocity and number density of the particles respectively, $K' = 6\pi\eta\rho\upsilon$, where η is particle radius, is the Stokes drag coefficient, $v_d = (l, r, s)$ and $\bar{x} = (x, y, z)$.

$$E=\varepsilon+\left(1-\varepsilon\right)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$$

which is constant, κ is the thermal diffusivity, ρ_s, C_s; ρ_0, C_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial v_d}{\partial t}+\frac{1}{\varepsilon}\left(\upsilon_d\cdot\nabla\right)\upsilon_d\right]=K'N\left(\upsilon-\upsilon_d\right), \tag{5}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\cdot\left(Nv_d\right)=0, \tag{6}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (5). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (5) for the particles.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$v=\left(0,0,0\right),\ T=-\beta z+T_0,\quad \rho=\rho_0\left(1+\alpha\beta z\right), \tag{7}$$

is an exact solution to the governing equations.

Let $\boldsymbol{v}$(u,v,w), θ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $\boldsymbol{v}$(0,0,0), temperature T, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ in temperature is given by

$$\delta\rho=-\alpha\rho_0\theta. \tag{8}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\frac{\partial v}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-g\frac{\delta\rho}{\rho_0}-\frac{1}{\kappa_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)\boldsymbol{v}+\frac{K'N}{\rho_0\in}\left(\boldsymbol{v}_d-\boldsymbol{v}\right), \tag{9}$$

$$\nabla\cdot v=0, \tag{10}$$

$$\left(E+b\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+\kappa\nabla^2\theta, \tag{11}$$

where $b=\dfrac{mNC_{pt}}{\rho_0 C_f}$ and w, s are the vertical fluid and particles velocity.

In the Cartesian form, equations (9)–(11) with the help of equation (8) can be expressed as

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t}=-\frac{1}{\rho_0}\frac{\partial}{\partial x}\left(\delta p\right)-\frac{1}{\kappa_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)u-\frac{mN}{\in\left(\frac{m\partial}{K'^{\partial t}}+1\right)\rho_0}\frac{\partial u}{\partial t}, \tag{12}$$

$$\frac{1}{\varepsilon}\frac{\partial v}{\partial t}=-\frac{1}{\rho_0}\frac{\partial}{\partial y}\left(\delta p\right)-\frac{1}{\kappa_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)v-\frac{mN}{\in\left(\frac{m\partial}{K'^{\partial t}}+1\right)\rho_0}\frac{\partial v}{\partial t}, \tag{13}$$

$$\frac{1}{\varepsilon}\frac{\partial w}{\partial t}=-\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\delta p\right)+g\alpha\theta-\frac{1}{\kappa_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)w-\frac{mN}{\in\left(\frac{m\partial}{K'^{\partial t}}+1\right)\rho_0}\frac{\partial w}{\partial t}, \tag{14}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{15}$$

$$\left(E+b\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+\kappa\nabla^2\theta. \tag{16}$$

Operating equation (12) and (13) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (15), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=\frac{1}{\rho_0}\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)\delta p-\frac{1}{k_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)\left(\frac{\partial w}{\partial z}\right)-\frac{mN}{\in\left(\frac{m\partial}{\mathrm{K}'\partial t}+1\right)\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right). \tag{17}$$

Operating equation (14) and (17) by $\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate $\delta\rho$ between equations (14) and (17), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 W\right)=-\frac{1}{\kappa_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)\nabla^2 W+g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\alpha\theta-\frac{mN}{\in\left(\frac{m\partial}{\mathrm{K}'^{\partial t}}+1\right)\rho_0}\frac{\partial}{\partial t}\left(\nabla^2 W\right), \tag{18}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

3. NORMAL MODE ANALYSIS

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$\left[w,\theta\right]=\left[W(z),\Theta(z)\right]exp\left(ilx+imy+nt\right), \tag{19}$$

where l and m are the wave numbers in the x and y directions, $k=(l^2+m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (19) in equations (18) and (16) become

$$\frac{n}{\varepsilon}\left[\frac{d^2}{dz^2}-k^2\right]W=-gk^2\alpha\Theta-\frac{1}{k_1}\left(\upsilon-\frac{\mu_c}{\rho_0}\nabla^2\right)\left(\frac{d^2}{dz^2}-k^2\right)W-\frac{mN}{\in\left(\frac{m\partial}{K'\partial t}+1\right)\rho_0}\left(\frac{d^2}{dz^2}-k^2\right)W, \tag{20}$$

$$\left(E+b\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+\kappa\left(\frac{d^2}{dz^2}-k^2\right)\Theta. \tag{21}$$

Equation (20) and (21) in non-dimensional form, become

$$\left[\left(1+\frac{M}{(1+\tau_1\sigma)}\right)\frac{\sigma}{\varepsilon}+\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W=-\frac{ga^2\delta^2\alpha\Theta}{\upsilon}, \tag{22}$$

$$\left[D^2-a^2-E_1P_r\sigma\right]\Theta=-\frac{\beta d^2}{\kappa}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{23}$$

where we have put

$a=kd,\sigma=\frac{nd^2}{\upsilon},\tau=\frac{m}{K'},\tau_1=\frac{\tau\upsilon}{d^2},E_1=E+b\varepsilon$ and $P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $P_r=\frac{\upsilon}{\kappa}$, is the thermal Prandtl number, B = 1 + b, is the suspended particles density parameter, $F=\frac{\mu_c}{\mu d^2}$, is the couple-stress parameter and $D^*=d\frac{d}{dz}=dD$ and dropping * for convenience.

Substituting $W=W^*$ and $\Theta=\frac{\beta d^2}{\kappa}\Theta^*$ in equations (22) and (23) and dropping * for convenience, we obtain

$$\left[\left(1+\frac{M}{(1+\tau_1\sigma)}\right)\frac{\sigma}{\varepsilon}+\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W=-Ra^2\Theta, \tag{24}$$

$$\left[D^2-a^2-E_1P_r\sigma\right]\Theta=-\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{25}$$

where $R=\dfrac{g\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar [1])

$$W=D^2W=\Theta=0 \text{ at } z=0 \text{ and } 1. \tag{26}$$

Then, we prove the following theorem:

THEOREM: If $R > 0$, $F > 0$, $B = 1 + b$, $b > 0$ and $\sigma = 0$, then the necessary condition for the existence of non-trivial solution (W, Θ) of equations (24) and (25) together with the boundary conditions (26) is that

$$R>\frac{4\pi^4F}{BP_l}.$$

PROOF: If the instability sets in stationary convection and 'principle of exchange of stability' is valid, the neutral or marginal state will be characterized by $\sigma = 0$. Thus the relevant governing equations (24) and (25) reduces to

$$\left[\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W=-Ra^2\Theta, \tag{27}$$

$$\left[D^2-a^2\right]\Theta=-BW, \tag{28}$$

together with the boundary conditions (26).

Multiplying equation (27) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{1}{P_l}\int_0^1 W^*\left(D^2-a^2\right)Wdz-\frac{F}{P_l}\int_0^1 W^*\left(D^2-a^2\right)^2Wdz=-Ra^2\int_0^1 W^*\Theta dz. \tag{29}$$

Taking complex conjugate on both sides of equation (28), we get

$$\left[D^2 - a^2\right]\Theta^* = BW^*. \tag{30}$$

Using equation (30) in the right hand side of equation (29), we obtain

$$\frac{1}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)Wdz - \frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 Wdz = \frac{Ra^2}{B}\int_0^1 \Theta^*\left(D^2 - a^2\right)\Theta dz. \tag{31}$$

Integrating term by term on both sides of equation (31) for an appropriate number of times by making use of boundary conditions (26), we obtain

$$\begin{aligned}&\frac{1}{P_l}\int_0^1\left(\left|DW\right|^2 + a^2\left|W\right|^2\right)dz + \\ &\frac{F}{P_l}\int_0^1\left(\left|D^2W\right|^2 + 2a^2\left|DW\right|^2 a^4\left|W\right|^2\right)dz = \frac{Ra^2}{B}\int_0^1\left(\left|D\Theta\right|^2 + a^2\left|\Theta\right|^2\right)dz.\end{aligned} \tag{32}$$

Since W and Θ satisfy $W(0) = 0 = W(1)$, $\Theta\ (0) = 0 = \Theta\ (1)$, we have by Rayleigh-Ritz inequality [9]

$$\int_0^1\left|DW\right|^2 dz \geq \pi^2\int_0^1 \left|W\right|^2 dz, \tag{33}$$

$$\int_0^1\left|D\Theta\right|^2 dz \geq \pi^2\int_0^1 \left|\Theta\right|^2 dz, \tag{34}$$

and

$$\int_0^1\left|D^2W\right|^2 dz \geq \pi^4\int_0^1 \left|W\right|^2 dz. \tag{35}$$

Further, multiplying equation (28) by Θ^* (the complex conjugate of Θ), integrating by parts each term of resulting equation on the right hand side for an appropriate boundary condition, namely $\Theta\ (0) = 0 = \Theta\ (1)$, it follows that

$$\begin{aligned}\frac{1}{B}\int_0^1\left(\left|D\Theta\right|^2 + a^2\left|\Theta\right|^2\right)dz &= \text{Real part of}\left(\int_0^1 \Theta^* W dz\right) \\ &\leq \left|\int_0^1 \Theta^* W dz\right|, \\ &\leq \int_0^1\left|\Theta^* W\right| dz, \\ &\leq \int_0^1\left|\Theta^*\right|\left|W\right| dz, \\ &\leq \int_0^1\left|\Theta\right|\left|W\right| dz,\end{aligned}$$

$$\leq \left(\int_0^1 |\Theta|^2 \, dz \right)^{1/2} \left(\int_0^1 |W|^2 \, dz \right)^{1/2} \text{. (by using Cauchy-Schwartz inequality)} \tag{36}$$

Thus, inequalities (34) can be written as

$$\frac{\pi^2 + a^2}{B} \left(\int_0^1 |\Theta|^2 \, dz \right)^{1/2} \leq \left(\int_0^1 |W|^2 \, dz \right)^{1/2}. \tag{37}$$

Combining inequalities (36) and (37), we obtain

$$\frac{1}{B} \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz \leq \frac{B}{\pi^2 + a^2} \int_0^1 |W|^2 \, dz. \tag{38}$$

Using inequality (33) in (38), we get

$$\frac{1}{B} \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz \leq \frac{B}{\pi^2 \left(\pi^2 + a^2 \right)} \int_0^1 |DW|^2 \, dz. \tag{39}$$

Thus, if $R > 0$, $F > 0$, $B = 1 + b$, $b > 0$, using the inequalities (35) and (38), the equation (32) becomes

$$\left(1 + a^2\right) I + \left[\frac{F}{P_l} \left(\pi^2 + a^2 \right) - \frac{RBa^2}{\pi^2 \left(\pi^2 + a^2 \right)} \right] \int_0^1 \left| D^2 W \right|^2 dz < 0, \tag{40}$$

where $I = \int_0^1 \left(|DW|^2 + a^2 |W|^2 \right) dz,$ is positive definite.

Therefore, we must have

$$\frac{F}{P_l} \left(\pi^2 + a^2 \right) < \frac{RBa^2}{\pi^2 \left(\pi^2 + a^2 \right)},$$

which implies

$$R > \frac{F}{BP_l} \frac{\pi^2 \left(\pi^2 + a^2 \right)^2}{a^2}, \tag{41}$$

Since the minimum value of $\frac{\pi^2\left(\pi^2+a^2\right)^2}{a^2}$ is $4\pi^4$ at $a^2 = \pi^2 > 0$, hence, we necessarily have

$$R > \frac{4\pi^4 F}{BP_l}, \tag{42}$$

which completes the proof of the theorem.

From physical point of view, the above theorem states that the onset of instability at marginal state in a couple-stress fluid heated from below permeated with suspended particles in porous medium cannot manifest as stationary convection, if the thermal Rayleigh number R, the couple-stress parameter F, medium permeability and suspended particles number density B, satisfy the inequality

$$R \leq \frac{4\pi^4 F}{BP_l} \tag{43}$$

4. CONCLUSION

The effect of suspended particles on thermal convection in couple-stress fluid in a porous medium has been investigated. From the above theorem, the main conclusions are as follows:

(i) The necessary condition for the onset of instability as stationary convection for couple-stress elastico-viscous fluid is

$$R > \frac{4\pi^4 F}{BP_l}.$$

(ii) The sufficient condition for non-existence of stationary convection at marginal state is

$$R \leq \frac{4\pi^4 F}{BP_l}$$

(iii) In the inequality (39), The thermal Rayleigh number $R > 0$, is directly proportional to the couple-stress parameter F. Thus, couple-stress parameter has stabilizing effect on the system as derived by Sharma and Sharma [7] and Kumar [13].

(iv) In the inequality (39), the thermal Rayleigh number $R > 0$, is inversely proportional to the suspended particles number density parameter B, which mathematically established the destabilizing effect of suspended particles number density parameter on the system as derived by Scanlon and Segel [4], Rana and Kumar [12], Rana and Sharma [14] and Kumar [13].

(v) The medium permeability has a destabilizing effect on the system as can be seen from inequality (39), which is an agreement with the earlier work of Sharma and Sharma [7], Rana and Kumar [12], Rana and Sharma [14] and Kumar [13].

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NOMENCLATURE

F	Couple-Stress parameter
P_l	Dimensionless medium permeability
g	Gravitational acceleration
$\mathbf{g}$	Gravitational acceleration vector
m	Mass of suspended particle
p	Pressure
K'	Stokes drag coefficient
N	Suspended particle number density
P_r	Thermal Prandtl number
v	Velocity of fluid
v_d	Velocity of suspended particles
k	Wave number of disturbance

Greek Symbols

β	Adverse temperature gradient
μ_c	Couple-Stress viscosity
ρ	Fluid density
μ	Fluid viscosity
ν	Kinematic viscosity
ε	Medium porosity
δ	Perturbation in respective physical quantity
θ	Perturbation in temperature
η	Radius of suspended particles
κ	Thermal diffusivity
α	Thermal coefficient of expansion