Improving the Design of Capacitive Micromachined Ultrasonic Transducers Aided with Sensitivity Analysis

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ABSTRACT

The paper presents the results of analysis performed to search for feasible design improvements for capacitive micromachined ultrasonic transducer. Carried out search has been aided with the sensitivity analysis and the application of Response Surface Method. The multiphysics approach has been taken into account in elaborated finite element model of one cell of described transducer in order to include significant physical phenomena present in modelled microdevice. The set of twelve input uncertain and design parameters consists of geometric, material and control properties. The amplitude of dynamic membrane deformation of the transducer has been chosen as studied parameter. The objective of performed study has been defined as the task of finding robust design configurations of the transducer, i.e. characterizing maximal value of deformation amplitude with its minimal variation.

Keywords: Microelectromechanical systems; capacitive micromachined ultrasonic transducer; finite element method; imaging technique; nondestructive testing; sensitivity analysis; Response Surface Method.

1. INTRODUCTION

Taking the advantage of both electronic and mechanical movable parts integrated in micro scale, microelectromechanical systems (MEMS) are successfully applied in biology, chemistry, measurement techniques, automotive industry, optics, etc [1,2,3,4]. Rapidly growing market for MEMS applications results in continuous engineering effort put into improvements of their designs [5,6,7]. On one hand there are performed laboratory tests dealing with new manufacturing techniques which are expected to allow for the improvements of characteristics of microdevices and therefore increase their performance. On the other hand virtual prototyping procedures are widely used to optimize design of MEMS in such a way that more robust solutions could be obtained still dealing with the same manufacturing processes and constraints on the quality of used construction materials. The activities which are mentioned first usually mean considerable costs spent on a new laboratory equipment and high-skilled technical staff. Incurred expanses seem to be necessary to achieve breakthrough in MEMS technology however there are reported successful design improvements for the microdevices by the applications of uncertainty and

sensitivity analyses as well as optimizations carried out with numerical models only [8,9]. Bringing both time and money savings a virtual prototyping stands for a complementary tool to the experiments with already manufactured MEMS components. The paper presents the results of sensitivity analysis performed to improve characteristics of chosen MEMS component, namely capacitive micromachined ultrasonic transducer (CMUT) [10,11,12,13]. CMUT composes a class of microdevices used for imaging technique and nondestructive testing (NDT). The structure of an exemplary CMUT and the principle of its operation are presented afterwards.

When a MEMS-type structure is designed the variation of geometry and properties of used materials should be considered to give reliable assessment of interesting operational characteristics [14,15,16,17]. It may happen that nominally the best configuration developed only with the results of deterministic analysis characterizes extremely high sensitivity of the MEMS performance to the scatters of geometry and material properties which are irreducible by nature. Uncertainty in MEMS exists irrespectively to applied manufacturing processes. It is of engineer's concern to find a solution which features possibly high operational performance and still stays robust for given uncertainties [18]. Numerical analyses performed to assess the sensitivity and uncertainty propagation can be aided with the Response Surface Method (RSM) [19,20,21,22]. The approximating techniques allow for additional time savings and give valuable results when properly applied.

Specificity of MEMS applications requires the multiphysics approach [23,24,25]. The following physical phenomena and domain interactions are most commonly applied: structural dynamics with both rigid and flexible solid components included, hydrostatics, hydrodynamics, fluid-structure interaction, thermodynamics, electrostatics, electrodynamics, electromagnetism, magnetism, friction phenomena, acoustics, piezoelectricity. Unlike for mechanical structures in macro scale, the proper modelling of microdevices often requires the introduction of more than two different physical phenomena. Some of them are responsible for the motion activation, e.g. electrostatics, thermal expansion, piezoelectricity as well as phenomena present in shape memory alloys and electroactive polymers. Multiphysics is also necessary to model correctly the principle of operation for transducers which, from their nature, have the ability of transferring the energy from one physical domain into another one. Finally multi domain approach allows to consider all phenomena which disturb the operation of MEMS device, i.e. which are responsible for energy dissipation. They all stand for immanent properties and must be selectively taken into account depending on the specificity of designed microdevice. The examples are air damping and acoustic emission.

The multiphysics approach can be applied using different methods of modelling. In general one can consider the following multi domain analyses [26,27,28]:

- analyses with simplified models,
- coupled analyses.

In case of analyses with simplified models different physical domains are introduced with their approximate substitutions [12,29,30]. There may be used approximating analytical formulas found with simulations or experiments. Then the relationships between domains, i.e. constitutive equations or any kind of coupling or energy transfer formulas, are identified according to established design of experiment (DOE). The examples are second order derivative equations describing the motion of microresonator or micromirror. These equations are identified for operational mode of vibration. Therefore they are valid when

mentioned above mode is considered to be the only one which occurs when microdevice deforms. The other application of simplified modelling is the introduction of additional discrete mechanical components e.g. dampers and springs which are intended to represent air damping and electrostatic field respectively. Resultant properties of additional components may be calculated accordingly to analytical equations which are widely known from the theory behind considered physical phenomena. The damping phenomena can also be indirectly taken into account with Rayleigh multipliers.

Coupled analyses, in turn, directly consider the presence of different domains by additional explicitly determined degrees of freedom (DOF) e.g. temperature, pressure, electric potential [11,13,25]. Required coupling and constitutive equations are introduced in their original forms. In general, it is expected that described group of multi domain approaches should lead to more accurate results. However longer computational time is usually required to yield the results. It is due to extended dimensionality of the problem, e.g. defined as the number of DOF and therefore also as the size of global system matrices being processed in finite element (FE) method. A hybrid approach is also possible. It appears when some of phenomena are indirectly introduced with approximating formulas into coupled analyses which take into account other physical domain too. A FE model of CMUT cell which is described in the paper stands for an example of mentioned approach. The contribution of air squeezing phenomenon in coupled analysis is considered with determined Rayleigh damping multipliers.

Depending on mutual relationships established between defined domains there may be applied weak coupling (also known as one-way coupling) or strong coupling (also defined as two-way/full coupling). A weak coupling means solving stated problem in one domain, or group of selected domains, and continuation of calculations with obtained results being mapped into other considered domains. The mapping procedure can be performed for example according to a mesh of grids defined for shared DOF. Final outcomes do not influence the analyses carried out before mapping procedure has been applied. A strong coupling in turn allows for simultaneous consideration of all physical domains including the whole set of related DOF. The second approach is more time consuming but when feasible and additionally properly applied it gives more accurate results.

The paper covers the following topics: section 2 presents the structure of CMUT and its principle of operation, section 3 describes elaborated FE model of one cell of CMUT including the explanation how the multiphysics approach has been applied, section 4 presents the workflow for carried out dynamic FE analyses, section 5 shortly describes introduced uncertain and design parameters dealing with geometry, material properties and control voltage, section 6 presents the results of sensitivity analysis, section 7 discusses possible improvements of CMUT properties, final section 8 summarizes the paper and presents concluding remarks.

2. CMUT-STRUCTURE AND PRINCIPLE OF OPERATION

The structure of one cell of CMUT is presented in Figure 1. It consists of electrostatically actuated micron-scale membrane suspended over a silicon substrate. Alternating voltage with DC bias generated in integrated electronic circuit is applied to power both top and bottom electrode and makes the membrane vibrate, as presented in Figure 2. Hence the ultrasonic wave can be generated and then emitted to monitored object. DC bias shifts a working point for the membrane. The introduction of static prestress is necessary to assure the reproducibility of membrane deformation accordingly to sin wave voltage supply. Proper control of the membrane deformation is otherwise not possible because the electrostatic force

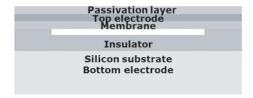


Figure 1 Structure of one cell of CMUT

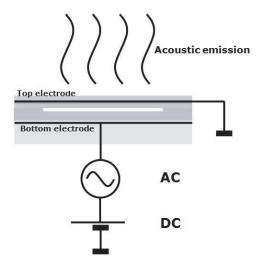


Figure 2 Power supply for CMUT cell

generated when the voltage, i.e. potential difference, is supplied always stands for the attraction force existing between powered electrodes. Deposited passivation layer prevents form chemical reactions with surrounding media in case of immersed application. A number of uniformly distributed CMUT cells can build a 1-D or 2-D array ultrasonic sensor used to speed-up the measurement process as well as to direct propagating wave. An exemplary application of 2-D CMUT array is presented in Figure 3. Integrated processor controls the voltages applied for successive cells.

There are known applications of CMUT technology for medical diagnosis as well as for NDT performed to assess the health of monitored mechanical structures [12,29]. Diagnostic information on the state of monitored object is derived from the parameters of reflected ultrasonic wave, e.g. time of flight, phase shift, amplitude. Larger and larger application area of CMUT encourages efforts to improve its performance and increase the quality of design.

Specificity of physical phenomena present in CMUT determines the necessity of the multiphysics application. In a general case there should be considered the following phenomena: structural dynamics to model basic mechanical properties, electrostatics for the membrane actuation, fluid-structure interaction to introduce the effect of fluid squeezing present in the area between membrane and substrate, and acoustics to model the propagation of ultrasonic wave in surrounding medium.

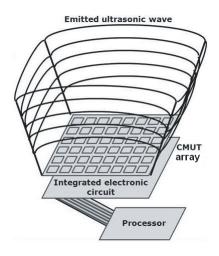


Figure 3 Emission of acoustic wave in 2-D CMUT array

3. FE MODEL OF CMUT, MULTIPHYSICS APPROACH

Parameterized FE model of CMUT cell has been elaborated in the ANSYS software [31]. Selected structural parts of the model are presented in Figure 4. The model has been used to study the influence of selected geometry and material properties on chosen characteristics of CMUT.

Apart from the structural parts, i.e. silicon membrane, silicon oxide support and silicon substrate which are all made of ANSYS/SOLID45, elaborated model consists of the following FE that allow for the consideration of multiphysics approach:

- 3-D squeeze film fluid elements ANSYS/FLUID136 used to model the phenomenon of
 squeezing of thin air film present between CMUT electrodes. Small gaps between parts
 moving one against another one cause the underpressure or overpressure which
 significantly change the dynamic properties of modelled structure.
- electromechanical transducers ANSYS/TRANS126 applied to represent established capacitors. Introduced elements are responsible for the generation of electrostatic force used to activate the membrane deformation.
- 3-D acoustic fluid elements ANSYS/FLUID30 introduced to create the spherical air space in which the CMUT cell is localized and acoustic waves can propagate. In the model there have been used fluid elements characterizing both 1-DOF and 4-DOF nodes. 4-DOF nodes are considered only within neighbouring area of the structural parts as they introduce three more DOF related to three-axis displacements, apart from

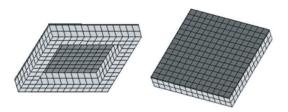


Figure 4 Suspended membrane of CMUT cell

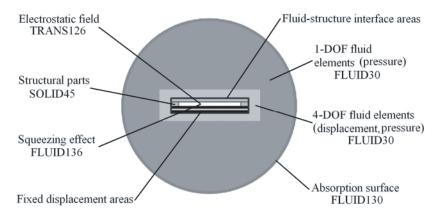


Figure 5 Physical domains in modelled CMUT cell

the pressure. To assure the savings on computational time a majority of covered space has been filled in with 1-DOF fluid elements.

 3-D infinite acoustic elements ANSYS/FLUID130 used to create an envelope to spherical air space. The elements simulate the absorbing surface and make the modelled acoustic space as it was extended to infinity.

The areas where respective kinds of FE have been applied are presented in Figure 5.

The width and the length of membrane equal $50\mu m$. The width of supporting posts equals $5\mu m$. The height of air gap equals $1\mu m$.

4. DYNAMIC FE ANALYSES

For parameterized FE model of one cell of CMUT the dynamic analyses have been performed accordingly to the workflow presented in Figure 6. The workflow has been implemented in MATHWORKS/MATLAB software [32]. Some operations on disk files have been aided with Microsoft Visual C# applications used to sped-up the calculations. Assumed set of input parameters is presented in section 5.

Presented workflow has been applied for each checked design configuration in order to perform the sensitivity analysis. First, after the parameterized model is prepared for given configuration of input parameters the prestress analysis is carried out. It is performed to simulate the effect of static deformation of membrane caused by DC bias which is applied to assure correct operation of CMUT. Then when a new operating point is established the modal analysis is performed to determine the Rayleigh damping multipliers for interesting mode of vibration i.e. fundamental mode of free vibrations which looks similar to the membrane's deformation at its regular operation. Found Rayleigh damping multipliers α and β are supposed to indirectly represent the squeezing effect present in the area between electrodes. They can be included into the matrix formulation defined for FE model where resultant global damping matrix C is expressed with the following formula [33]:

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \tag{1}$$

where M and K are global mass and stiffness matrices respectively. Finally, the static prestress analysis is repeated with already included Rayleigh damping multipliers and then followed with one-point frequency response analysis. It is performed to find the

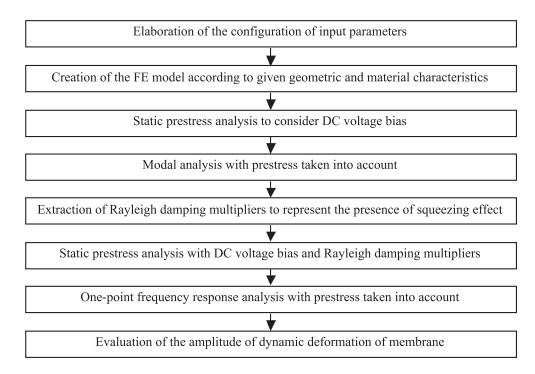


Figure 6 Workflow used for calculations of dynamic properties of CMUT cell

amplitude of dynamic deformation for assumed operation frequency 5MHz for which CMUT cell is designed. An exemplary view of membrane deformation and cross section of modelled space with pressure distribution found for operational mode shape is presented in Figure 7.

The height of air gap has been deliberately increased for shown exemplary case to be able to check whether the deformation of membrane is correct and pressure distribution looks reasonably. However while the sensitivity analysis all the input parameters have kept their assumed values precisely.

When the correctness of elaborated workflow was checked the analysis of mesh convergence has been performed to chose the compromise size of FE used in the CMUT model. There have been carried out dynamic simulations for nominal design configuration with the following sizes of solid FE used to construct the structural parts: $10\mu m$, $5\mu m$, $2\mu m$, $1\mu m$ and $0.5\mu m$. The sizes of remaining fluid FE have been automatically set by built-in smart meshing procedure available in the ANSYS software. The results of harmonic analyses performed for the frequency range 3MHz-10MHz are presented in Figure 8. The computational time needed to solve analysed cases with the same PC configuration are gathered in Table 1.

For the CMUT cell at its normal operation, as mentioned above, the frequency 5MHz has been chosen for the supplied control voltage. A case defined by the size $1\mu m$ has been arbitrary chosen as acceptable compromise between the quality of results and required computational time. Assumed choice seems to give reasonable results with acceptable calculation effort.

The value of operational frequency has been deliberately chosen so that it differs from the resonance frequency related to the interesting normal mode. Such an approach



Figure 7 Membrane deformation and pressure distribution for CMUT cell while its operation

is expected to help in a search for a robust configuration of CMUT cell. The less difference between the frequency of control voltage and the natural frequency is the more sensitive the amplitude of membrane deformation to the variation of resonance frequency due to the uncertainty propagation is. However the deformation amplitude takes its desired maximal value for the resonance peak. Hence the search for both

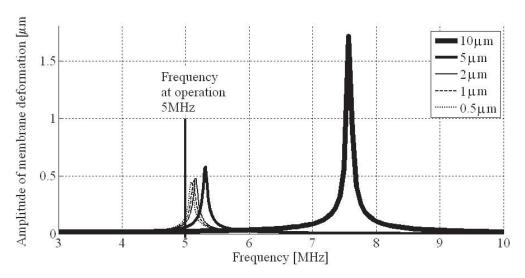


Figure 8 Convergence of amplitude of dynamic deformation of membrane

Table 1 Time consumption for simulations of dynamic properties of CMUT cell

Case-size of FE used for	Required computational time for a single	
structural components	frequency step in harmonic analysis	
10μm	30s	
5μm	48s	
2μm	73s	
1µm	167s	
0.5µm	25min	

efficient and robust design should be performed within the neighbouring area of localized resonance peak.

5. INPUT PARAMETERS OF DYNAMIC ANALYSES

For the analysis of dynamic deformation of membrane a set of 12 design and uncertain parameters has been established. Allowed ranges of their variations are presented in Table 2.

There have been assumed two design parameters: the membrane thickness T and the height of air gap H. Allowed ranges of their variations stand for the interesting domain in

Table 2 Input uncertain and design parameters

No.	Description	Symbol	Nominal value	Range of variation / uncertainty
1	Thickness of membrane (design parameter)	Т	1μm	1-1.5μm
2	Height of gap (design parameter)	Н	1µm	1-2µm
3	Young's modulus of silicon	E1	150GPa	±3% (±4.5GPa)
4	Poisson's ratio of silicon	n1	0.17	±3% (±0.0051)
5	Density of silicon	D1	2332kg/m ³	$\pm 3\% \ (\pm 70 \text{kg/m}^3)$
6	Young's modulus of silicon oxide	E2	73GPa	±3% (±2.19GPa)
7	Poisson's ratio of silicon oxide	n2	0.17	±3% (±0.0051)
8	Density of silicon oxide	D2	2200kg/m^3	$\pm 3\% \ (\pm 66 \text{kg/m}^3)$
9	Density of air	D3	1.225kg/m ³	$\pm 3\% \ (\pm 0.0368 \text{kg/m}^3)$
10	Viscosity of air	u3	18.3×10⁻⁶Pa⋅s	±3% (±549×10 ⁻⁹ Pa·s)
11	Voltage (AC amplitude and DC bias)	V	30V	±1% (±0.3V)
12	Frequency	f	5MHz	$\pm 10^{-6} (\pm 5 \text{Hz})$

which improved design configurations have been searched for. Remaining parameters describe the uncertainty of chosen material properties and applied control voltage. All included material properties characterize an arbitrary assumed $\pm 3\%$ -uncertainty related to their nominal values. In case of the control voltage narrower ranges on uncertainty have been assumed as rational ones, i.e. $\pm 1\%$ for the amplitude of voltage and only $\pm 10^{-6}$ for its frequency as feasible in electronic generators. All input parameters are treated as not correlated.

6. SENSITIVITY ANALYSIS

The sensitivity analysis has been performed for the quantitative assessment of the influence of all defined input parameters on the studied amplitude of membrane dynamic deformation Δ . The values of sensitivities S have been calculated using amplitudes Δ successively found for uniformly sampled domains of input parameters separately and accordingly to allowed ranges of their variation. Therefore determined S stands for the worst case scenario in which the maximal differences in studied characteristics Δ are considered. The sensitivities are already normalized to the nominal deformation amplitude $\Delta_{NOM} = 0.1215 \mu m$. The formula which has been used to assess the sensitivities takes the following form:

$$S_n = \frac{\left(\max_i (\Delta_{i,n}) - \min_i (\Delta_{i,n})\right)}{\Delta_{NOM}} \cdot 100[\%]$$
 (2)

where: n denotes the index of input parameter $(1 \le n \le 12)$, i stands for the index of current sample (assumed that $1 \le i \le 11$), $\Delta_{i,n}$ means the amplitude of membrane dynamic deformation Δ calculated for n-th input parameter with its i-th configuration. Found sensitivities are presented in Figure 9.

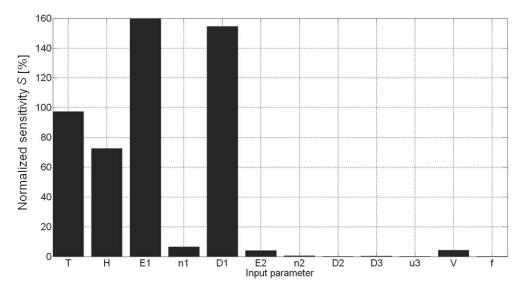


Figure 9 Normalized sensitivities

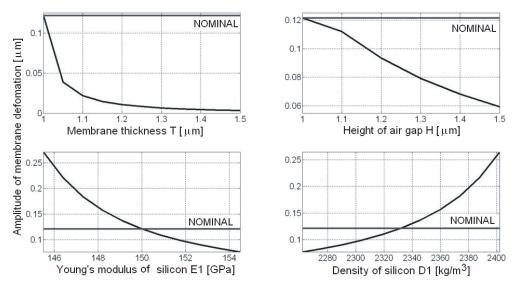


Figure 10 Relationships between the most influential input parameters and output parameter

On the basis of calculated sensitivities one can easily find the most influential parameters: the Young's modulus E1 and the density D1 of silicon ($S_3 = 160.0\%$, $S_5 = 154.5\%$), the membrane thickness T ($S_1 = 97.3\%$) and the gap height H ($S_2 = 72.7\%$). The cumulative sensitivity calculated for four mentioned above parameters stands for 96.9% of the total sum of sensitivities determined for all input parameters. Yielded results justify the choice of design parameters T and H as they significantly contribute in the variation of amplitude Δ . The parameters n2, D2, D3, u3 and f seem to characterize negligible influence on Δ . The relationships between four the most influential parameters and the amplitude Δ are shown in Figure 10.

Found relationships are nonlinear. Therefore the RSM has been applied to perform more accurate investigation on the sensitivities especially taking into account the interaction terms. A three-level full factorial DOE has been used for seven most influential parameters: T, H, E1, n1, D1, E2, V. It has resulted in 2187 checked design samples. There have been assumed the following 135 regressors included in the approximation function: a constant, 7 linear terms, 120 interaction terms (from the second up to the seventh order) and 7 quadratic terms. The coefficients of regressors have been determined with the least square method. They are presented in Figure 11. Figure 12 shows 17 the most influential regressors which characterize coefficients greater than 0.005.

The values of regressor coefficients confirm previous observations on the relationships between input and output parameters. Apart from the great values of coefficients related to the linear terms (T, H, E1, D1) there have been revealed influential higher order terms included in the approximation. Considerable nonlinearity of the relationship between T and Δ (seen in Figure 10, the upper left plot) is clearly confirmed by the high significance of the regressor T². Moreover there is observed the strong influence of interaction terms, especially those ones containing the thickness of mirror: TH, TE1, TD1, THE1, THD1, etc. Presented results of the sensitivity analysis have been finally used to conclude about possible design modifications to assure enough high amplitude of membrane deformation with its reduced variation as expected for a robust configuration.

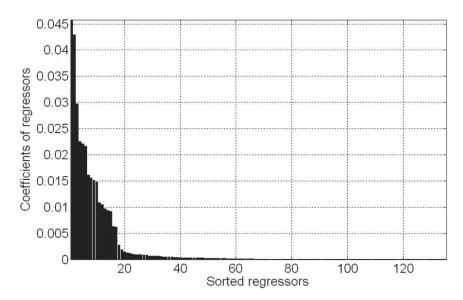


Figure 11 All calculated regressor coefficients

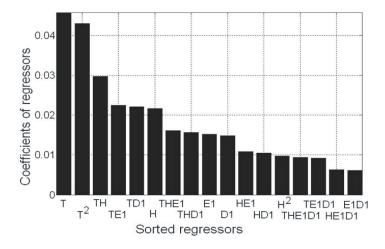


Figure 12 Coefficients of the most influential regressors

7. IMPROVING CMUT DESIGN

Chosen 9 uniformly distributed design configurations have been checked to discuss possible improvement of the dynamic properties of modelled CMUT cell. The combinations of pair T, H have been determined accordingly to the assumption that each parameter can take three possible values from allowed range of variation: $T \in \{1\mu m, 1.25\mu m, 1.5\mu m\}$, $H \in \{1\mu m, 1.5\mu m, 2\mu m\}$. On the basis of results obtained for used DOE (three-level full factorial DOE, 2187 samples) for each of 9 design configurations the variation of membrane deformation Δ has been assessed. The maximal and minimal values of parameter Δ have been found for remaining 5 uncertain parameters E1, n1, D1,

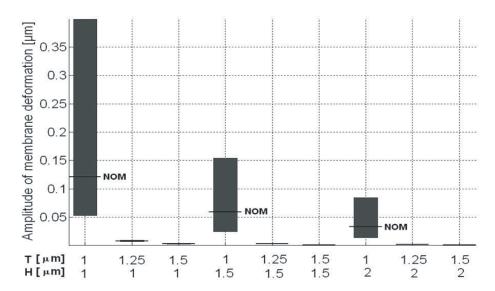


Figure 13 Nominal values and scatters of membrane deformation

E2, V, i.e. amongst 243 samples for each design accordingly to applied DOE. Figure 13 presents the nominal values and the scatters of Δ calculated for each checked design configuration.

A huge difference on the variation of parameter Δ has been found while checking the design domain. When only the reduction of variation of membrane deformation Δ is of concern one can find the designs characterizing $T=l\mu m$ as unacceptable. It is clearly seen that for these cases assessed variations of Δ , i.e. the differences between found maximal and minimal values of Δ while varying 5 uncertain parameters, exceed their nominal values significantly. When $T=l\mu m$ the ranges of variation equal respectively: 284.6% (H = $1\mu m$), 217.6% (H = $1.5\mu m$), 213.0% (H = $2\mu m$) of related nominal values of Δ . Mentioned above design configurations are not robust for introduced uncertainty. However they may be deliberately applied when the CMUT performance is the most important criterion. In such an approach the configuration $T=H=1\mu m$ leads to the highest membrane deformation found for given input parameter domain. The nominal and the extreme values of Δ equal respectively: $0.1215\mu m$, $0.0534\mu m$ and $0.3992\mu m$. Figure 14 presents more robust configurations when $T \geq 1.25\mu m$. Found ranges of variation of Δ related to their nominal values are presented in Table 3.

All obtained relative ranges of variations are comparable for cases when $T \geq 1.25 \mu m$. It means that the CMUT configuration $T=1.25 \mu m$, $H=1 \mu m$ can be chosen as the best one because of its highest performance observed for all considered designs. The final conclusion however could be that the best designs can be found for $H=1 \mu m$ and for the membrane thickness taken from the interval $1 \mu m \leq T \leq 1.25 \mu m$. Hence the choice should be made depending on included weighting factor allowing to establish the compromise between the performance and the robustness of CMUT cell.

8. SUMMARY AND CONCLUDING REMARKS

The paper presents the results of analyses performed to elaborate guidelines for improving the dynamic characteristics of CMUT. The FE model of a one-cell CMUT has been created

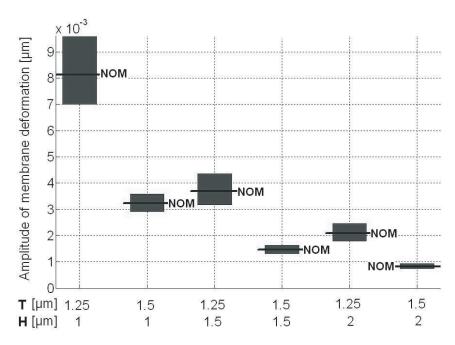


Figure 14 Design configurations characterizing increased robustness

and parameterized to allow for the sensitivity analysis. The multiphysics approach has been applied to model several physical phenomena present in CMUT: structural dynamics, electrostatics, air damping and acoustics. The amplitude of dynamic deformation of silicon membrane elaborated for chosen operational frequency 5MHz has been chosen as the parameter of interest. There have been introduced the following design and uncertain parameters: the geometric and material properties as well as the characteristics of applied control voltage. Before any inference on design improvement has been made the convergence analysis had been performed to choose the correct mesh for the FE model. There has been achieved satisfying compromise between quality of results and computational time.

The sensitivity analysis has been performed for the quantitative assessment of the influence of all defined input parameters on studied parameter. It turns out that the most influential input parameters are the membrane thickness, the height of air gap, the Young's

Table 3 Ranges of variation related to the nominal membrane deformation

Design config	uration	Range of variation of amplitude	
Thickness of membrane	Height of air gap	of membrane deformation	
T [μm]	H [μ m]	related to its nominal value $[\%]$	
1.25	1	31.9	
1.5	1	20.7	
1.25	1.5	32.4	
1.5	1.5	20.9	
1.25	2	32.5	
1.5	2	20.9	

modulus and the density of silicon. The results obtained with axial DOE have been confirmed the those ones yielded for three-level full factorial DOE. The nonlinearities of relationships between input and output parameters have been identified with the coefficients of interaction terms included in elaborated metamodel.

On the basis of performed analysis one can state that the choice of correct design of CMUT always deals with the compromise between its performance (efficiency) and robustness. As shown in the paper there are possible both performance-based or robustness-based criteria used to search for the best configuration. The introduction of weighting factor allows to control effectively the balance between contributions of mentioned criteria. For elaborated FE model of CMUT cell there have been proposed different feasible design configurations applicable when either increased performance or robustness is expected.

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