Analysis of Wave Propagation in a Homogeneous Dielectric Crystal

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ABSTRACT
Wave propagation through crystal was analysed using wave equation in conjunction with pseudo-dielectric function. Considered in the analysed were the patterns of propagated wave for UV, Visible and near Infrared for a particular value of dielectric constant of the crystal. The relationship between wavenumber and refractive, optical conductance and absorption co-efficient were analysed. The effects of focusing angle on the crystal for various wave-length regions were also considered.

Keywords: Dielectric constant, crystal, propagation, refractive index, electromagnetic wave, pseudo-dielectric function, wave number

1. INTRODUCTION

Propagation of wave through crystal material has been of interest in quest for good materials for optoelectronics, solar cell etc. The early work of Feit and Fleck (Feit, 1978, Fleck, 1978) induced a significant impact on the use of beam propagation study in the understanding of the optical properties of crystals. The applicability of this technique in the assessment of the optical response of crystals to the propagated wave had lead to great extent the understanding of technological importance of crystal. Roey et al 1981 and Thylen in 1982 demonstrated in their work the efficacy of beam propagated and modeled the concept in the study of integrated optics.

Electromagnetic wave incident on a crystal at normal or oblique on the surface of a crystal experiences attenuation as the wave propagates through it (Thylen and Lee 1992). The extent of the attenuate suffered by the wave depends on the solid state properties of the crystal such as the dielectric function of the crystal which is related the refractive index in which properties of solid state crystal materials strongly depend on (Chatterjee, 2003).

As early as 1968, Veselago, a soviet Physicist analyzed theoretically the electromagnetic properties of media in which the real part of the magnetic permeability μ and the electric permittivity ε were both negative (Veselago, 1968) even before experimental measurements of refractive index using various techniques and methods were still on course, Stanley and Meeten respectively (Stanley, 1989 and Meeten, 1986) with the interest geared towards

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influence of this property on the behavior of the propagated wave patterns. The dielectric constant *k* with uniform magnetic permeability μ is periodic (Zhang and Satpathy, 1990) and the influence of refractive index on the propagated wave in one dimensional inhomogeneous deterministic media evaluated (Diamant and Fernandez-Guasti, 2009). The intermediate case, where the refractive index varies on a small scale has been considered (Ugwu et al, 2007) and a useful strategy has been used to solve the differential equations for a large number of thin homogenous crystal layers and then considered where the permittivity approaches the desired value (Abeles, 1950).

In this work, wave propagation in a homogenous dielectric crystal is analysed using general plane wave equation with a homogenous dielectric constant obtain using pseudo-dielectric functions of the crystals obtained using measured value of the refractive index. This was considered over a wide wavelength range of radiation in order to ascertain the behaviour on neglecting the magnetic permeability of the crystal. In the past, researches have use beam propagation technique involving vectorial effect on propagation, but in this case, scalar wave equation was used (Martin et al, 1994 and Liu et al, 1992) which enables one to handle propagated wave without much difficulties by neglecting the polarization effect of the crystal and the concept of Brewster's angle (Born et al, 1980 and Elshazly-Zaghloul et al, 1983).

2. ANALYTICAL FORMULATION AND SOLUTION OF THE PROBLEM

A system of equation that tends to describe the propagation of electromagnetic waves in crystal medium consists of the Maxwell equations according Peschanskii and Toryanik (Peschanskii et al 1998). As a result, we start with general wave equation which is the basic concept used for the analysis. Wave equation is a second order linear partial differential equation that describes the propagation of wave in various media. So far, we consider a periodic crystal structure and use scalar wave approximation where the wave vector nature of the electromagnetic wave is neglected. We consider here the scalar wave equation for which the magnetic permeability μ is uniform throughout and the dielectric constant $\in$.

From the wave equation given as

$$\nabla^2\Psi(r) = \frac{\in}{c^2}\frac{\partial^2\Psi(r)}{\partial t^2} \tag{1}$$

We have

$$\nabla^2 E_x = \frac{\in}{c^2}\frac{\partial^2 E_x}{\partial t^2} \tag{2}$$

and

$$\nabla^2 B_y = \frac{\in}{c^2}\frac{\partial^2 B_y}{\partial t^2} \tag{3}$$

where E_x and B_y are the electric and magnetic fields component respectively.

Considering the direction of propagation, for free space, the solutions are generally given as

$$E_x = E_0 \, expi(kx - \omega t) \tag{4}$$

$$B_y = B_0 expi(ky - \omega t) \tag{5}$$

In the case of propagation of wave through a dielectric crystal medium with speed v, wave number k of the propagation wave becomes

$$k = \sqrt{\in} / c \;\; \omega = 2\pi / \lambda \sqrt{\in} = 2\pi / \lambda n \tag{6}$$

From

$$\in(\omega) = \in r(\omega) + \in i(\omega) \tag{7}$$

which is the pseudo-dielectric function, the value of the dielectric constant is obtain in term of the refractive index, n and extinction co-efficient, K both for real and complex parts respectively.

$$\in r = n\ 2 - K\ 2 \tag{8}$$

$$\in i = 2nK \tag{9}$$

where n and K are measurable experimentally.

With this it is sufficient to solve equation (1) in term of relationship between electric and magnetic field components. Introducing the dielectric feature of the crystal as

$$B_y = \sqrt{\mu \in} E_x = \sqrt{\mu} n E_x \tag{10}$$

The wave propagating through the crystal becomes

$$\Psi(z) = \Psi_0 \, \mathrm{expi}\left[\left(\frac{2\pi}{\lambda}\sqrt{\in}\right) z - \omega t\right] \tag{11}$$

From equation (11), the result of the wave pattern depicting the propagated wave behavior were obtained. This was made possible by considering the crystal to be uniaxial crystal with the propagated wave linearly polarized. The propagated wave was considered within the range of UV, visible and near infrared in which the dielectric characteristic of the crystal was chosen to reflect the solid state property of the crystal. Considered also were the normal and oblique incident of the propagated wave and the optical conductivity of the crystal (Centin et al, 2007 and Gray, 1982)

$$\delta\ op = \alpha nc / 4\pi \tag{12}$$

where α is the Absorption coefficient defined as $\alpha = \dfrac{4\pi K}{\lambda}$, K is the extinction coefficient of the crystal.

3. RESULTS AND DISCUSSION

In fig. 1 to fig. 6, the propagating wave pattern was presented both for real and complex parts respectively for the three regions UV, Visible and near Infrared in the graphs as a function of the crystal distance, z. fig. 7 and fig. 8 depicted the graphs of the propagated wave number

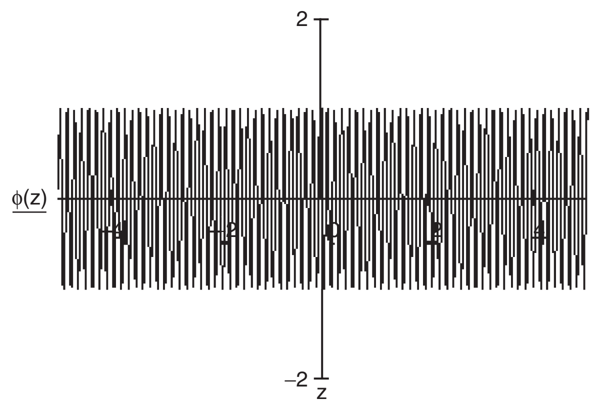

Figure 1 Complex part of the propagated pattern in the crystal within UV region.

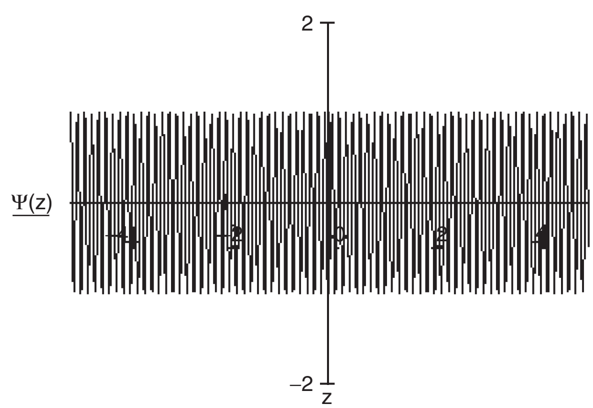

Figure 2 Real part of the propagated pattern in the crystal within UV region.

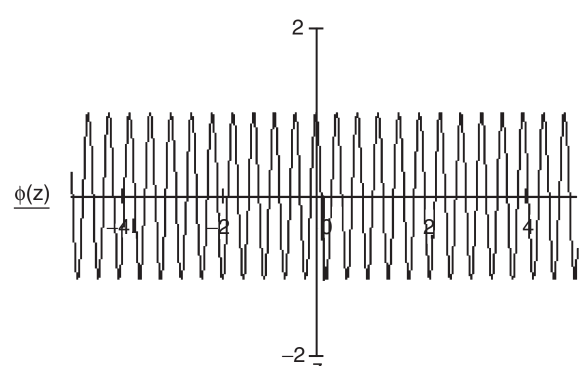

Figure 3 Complex part of the propagated pattern in the crystal within optical region.

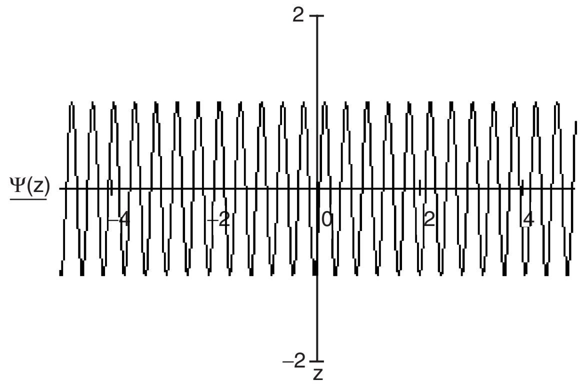

Figure 4 Real part of the propagated pattern in the crystal within UV region.

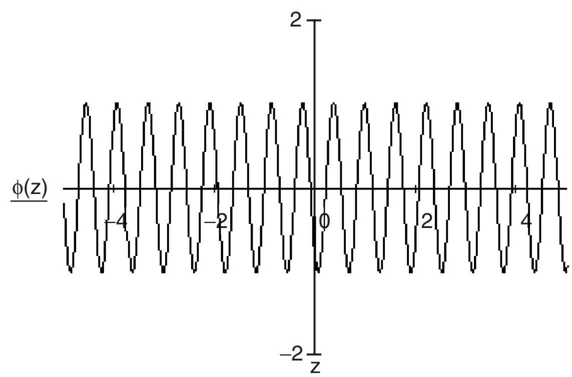

Figure 5 Complex part of the propagated pattern in the crystal within Infrared region.

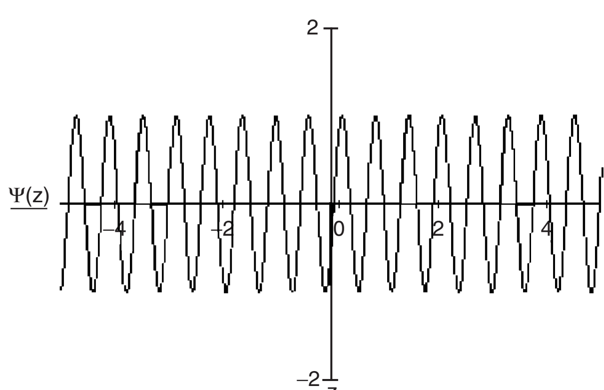

Figure 6 Real part of the propagated pattern in the crystal within Infrared region.

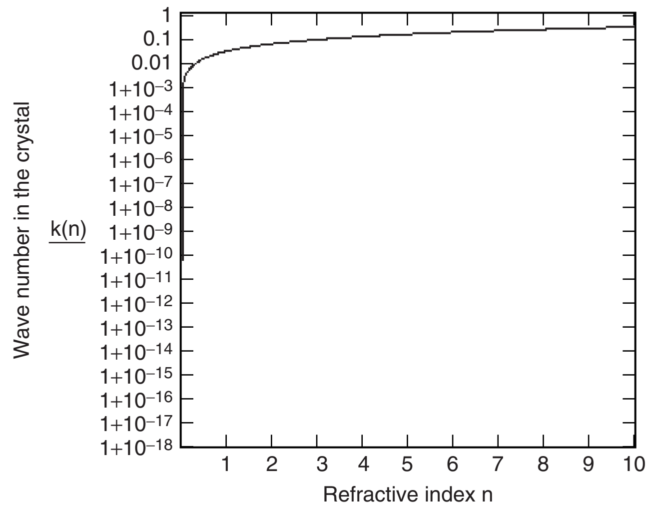

Figure 7 The wavenumber in the crystal k(n) against the refractive index for UV region.

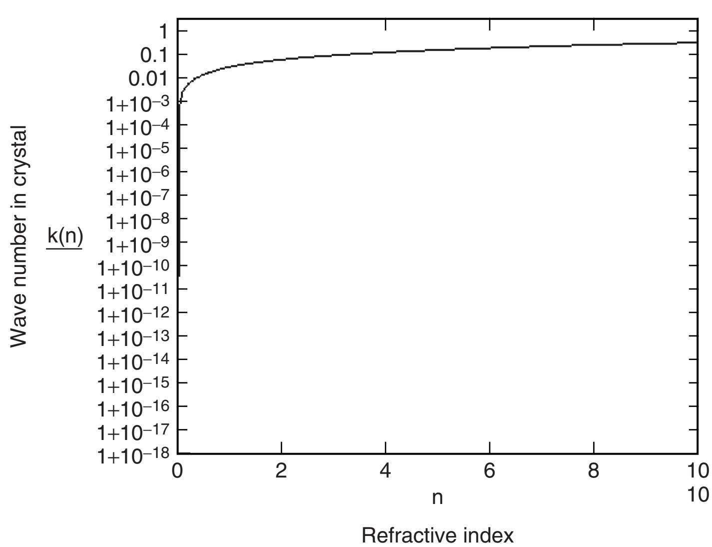

Figure 8 The wavenumber in the crystal k(n) against the refractive index for visible region.

in the crystal against the refractive index. Presented in the result were graphs of optical conductance as a function of absorbance co-efficient for given two refractive indices if the crystal 1.58 and 2.58 respectively in fig. 9 and fig. 10. The field orientation pattern in the crystal for various regions as defined earlier were shown in fig. 11 to fig. 16 at angle 45^{o} and 90^{o} respectively.

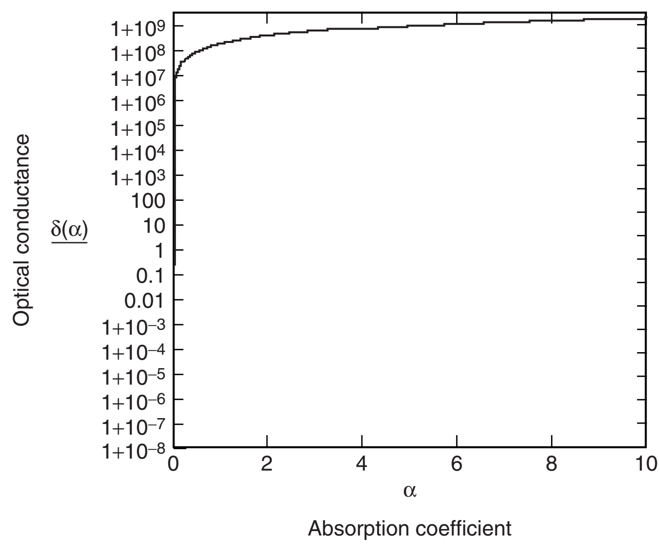

Figure 9 Optical conductance against absorption co-efficient when n = 2.58.

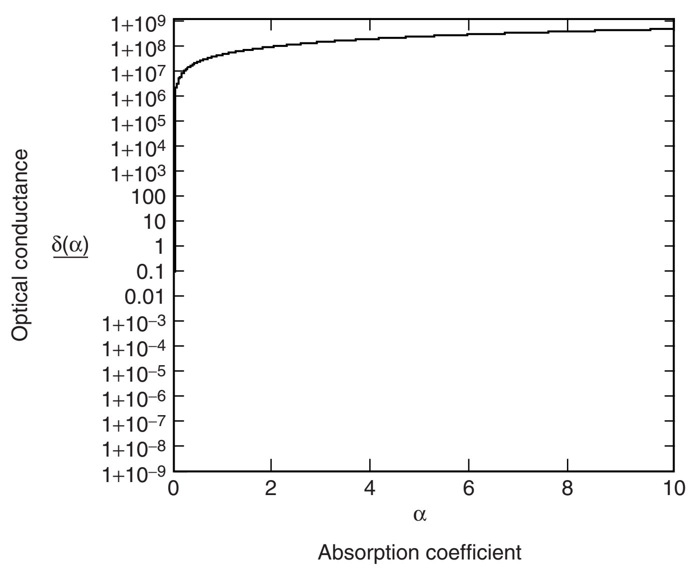

Figure 10 Optical conductance against absorption co-efficient when n = 1.58.

The wave pattern as presented in fig. 1 to fig. 6 showed a high frequency profile in the UV region for both real and complex parts, less in the Visible region and least for near Infrared region. These were determined by fitting by fitting in a value for dielectric constant of the crystal as obtained from the outcome of equations (8) and (9) respectively originating from equation (7) for both real and complex parts.

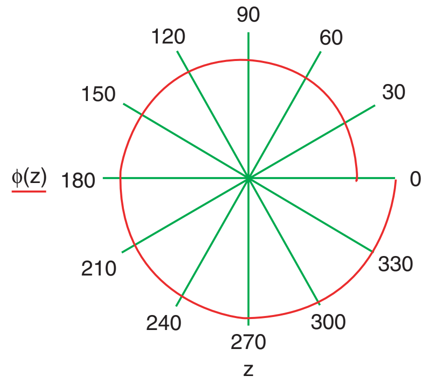

Figure 11 The field orientation in the crystal when n = 2.58 at θ = 45° for UV wavelength range (250 nm).

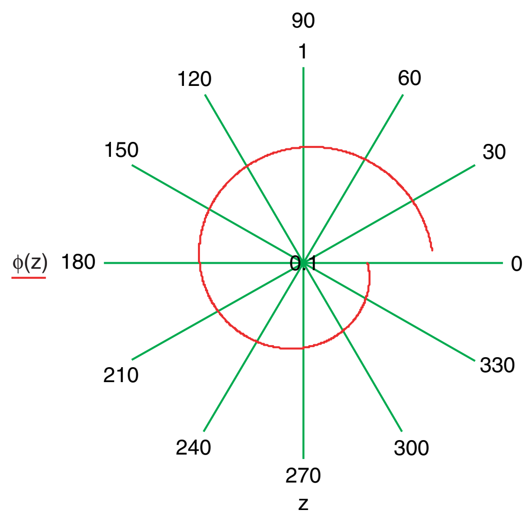

Figure 12 The field orientation in the crystal when n = 2.58 at θ = 90° for UV wavelength range (250 nm).

It is worth noting that figures 7, 8, 9 and 10 have the same look due the effect of refractive index on the solid state properties of any given crystal in relation to the optical properties of which energy band gap as $\alpha^2 = h\nu - Eg$ is one of the determinant.

It was also seen in figures 11 to 16 the pattern assumed by the orientation of the propagated wave through the crystal. In fig. (11) and fig. (12) the spiral pattern tended to increase in the diameter as it completes a circle for θ = 45° in UV region (250 nm) while it

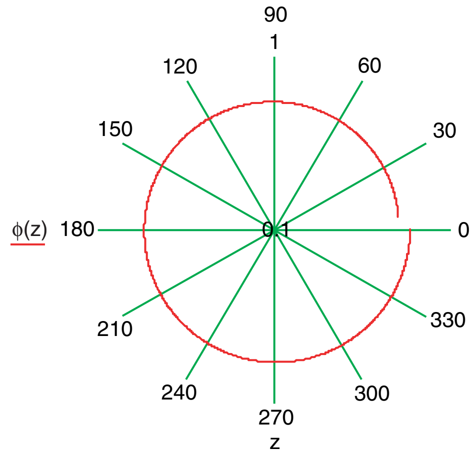

Figure 13 The field orientation in the crystal when n = 2.58 at θ = 90° for visible wavelength range (600 nm).

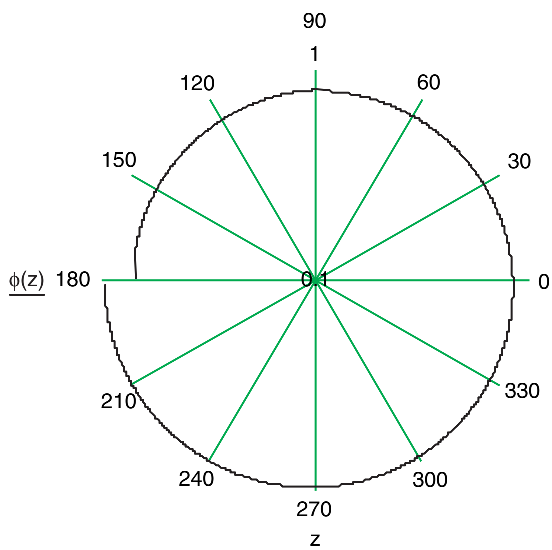

Figure 14 The field orientation in the crystal when n = 2.58 at θ = 45° for visible wavelength range (600 nm).

was opposite for θ = 90° of the counterpart. For visible region, the patterns were different altogether as observed in fig. (13) and fig. (14) respectively while for near infrared with angle of incident θ = 45°, the pattern started from the origin and increase radially on completing a circle as in fig. 15. For fig. 16 the radial pattern decreased. This behaviour could be variation in the phase velocity in the crystal medium which is strictly depended on the dielectric constant and the polarization effect which was not taken into consideration.

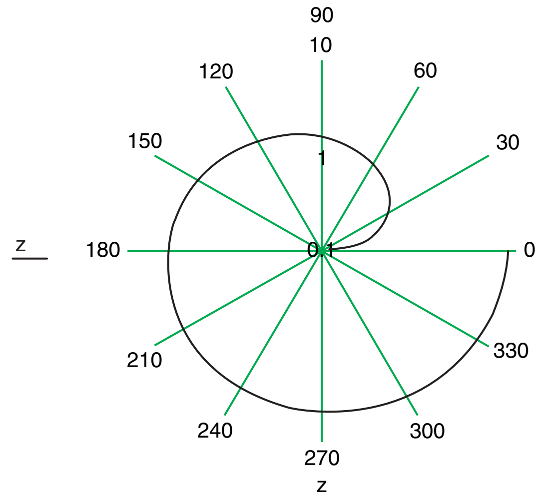

Figure 15 The field orientation in the crystal when n = 2.58 at θ = 45° for infrared wavelength range (900 nm).

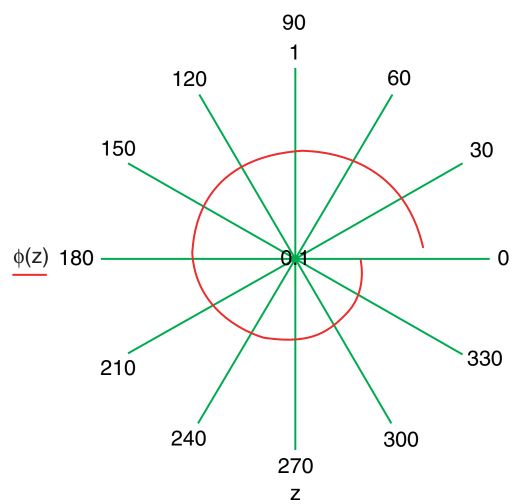

Figure 16 The field orientation in the crystal when n = 2.58 at θ = 90° for infrared wavelength range (900 nm).

4. CONCLUSION

In the present work, the analysis of the propagation of electromagnetic wave in a homogenous dielectric crystal using the general wave equation had been carried out. The relationship between the refractive index of the dielectric crystal and electric field was established. From the work it seen that the behaviour of electromagnetic wave propagation through crystal depended on the solid state properties of the crystal which manifested in the three regions of electromagnetic wave considered i.e. UV, visible and near Infrared region.

REFERENCES

[1] M. D. Feit and J. A. Fleck 1978. Appl. Opt. 17; 3990–3998.
[2] M. D. Feit and J. A. Fleck 1978. Appl. Opt. 18; 2843–2851.
[3] J. van Roey and J. van der Donk 1981. J. Opt. Soc. Am. 71 803–810.
[4] L. Thylen and C. M. Lee 1982. Opt. Quantum Electron 15 433–439.
[5] S. Chatterjee 2003 Physica Scripta 57, 234.
[6] V.G. Veselago 1968. The electrodynamics of substances with simultaneously negative values of ε and μ. Sov. Phys. Usp 10, 509–514.
[7] G.E. Stanley 1989. Refractometer; Basic Principles. Bellingham and Stanley Ltd.
[8] G.H. Meeten (Ed) 1986. Optical properties of Polymers. Elsevier, London.
[9] Ze-Zhang and Sashi Satpathy 1990. Physical Review let. Vol. 65 No. 21 2650–2653.
[10] Diamant and Femandez Guasti, 2009. J. Opt. A Pure Appl. Opt. 11, 045712, 1–8.
[11] E. I. Ugwu, P. C Udah and G. A. Agbo 2007. J. Appl. Sc. 7(4) 570–574.
[12] L. Abeles 1950 Ann. Phys. F05 706–84.
[13] O. J. F. Martin, A. Deveus and C. Girard 1994 J. Opt. Soc. Am. Vol.11 No.3 1073–1080.
[14] J. M. Liu and L. Gomelsky 1992. J. Opt. Soc. Am. A9 1574–1585.
[15] M. Born and E. Wolf 1965. Principle of Optics Oxford: Pergamon Press.
[16] M. Elshazly-Zaghloul and R.M.A. Azzam 1983. J. Opt Soc. Am. 72: 657–661.
[17] V.G. Peschanski and Toryanik D. A. (1998) AIP vol. 40 No. 8 pp. 1272–1280.
[18] S.A. Cetin and I. Hikment 2007 AJP 978–0–7354.
[19] D.E. Gray 1982 American Institute of Physics Handbook, 3rd Edition McGraw Hill Book Co N.Y.