Experimental Study of Impact on Composite Plates with Fluid-Structure Interaction

Young W. Kwon, Angela C. Owens, Aric S. Kwon, Jarema M. Didoszak
Department of Mechanical & Aerospace Engineering
Naval Postgraduate School
Monterey, California 93943

ABSTRACT
The transient dynamic response of composite structures under water is affected by Fluid Structure Interaction (FSI), which results in an added mass effect as well as damping. Because the density of composites is comparable to that of water, the added mass effect becomes even more critical to the transient dynamic response of composites in water. In this study, an experimental testing set-up was designed and fabricated, and testing was conducted to investigate FSI effects on composite laminate plates immersed in fluid and subjected to impact loading. Square composite laminates made of carbon fiber weave and vinyl ester resin were subjected to impact loading using a specially developed vertical drop-weight testing machine. The composite samples were fitted with gages to provide time-history on strains and impact forces generated during impact. Impact tests were performed on four-side clamped laminate plates in air-backed wet, water-backed wet, and dry environments. The results showed non-uniform effects on transient responses of wet composites with FSI. Generally, wet impacts on composite plates increased both transient impact forces and strains significantly compared to dry impacts under the same impact mass and velocity condition. The findings of this study will provide a better understanding for use of composite materials in underwater structural applications where impact loading is expected.

Keywords: impact, composites, fluid-structure interaction

1. INTRODUCTION

The growing use of composites in ship masts, superstructures, deck grates, piping, ducting, rudders, propellers, stacks, and various submarine structures requires extensive modeling and testing to help designers, builders and operators better understand composite response [1]. These materials are subjected to a wide spectrum of loads during manufacturing and service life. Dynamic loadings, in particular, impact type events, represent a serious design concern for use of composites since composite structures are more susceptible to impact damage than similar metallic structures which are more ductile in nature and can typically absorb large amounts of energy without failure [2]. Furthermore, the damage in composites from impact can go undetected even when the mechanical properties may be drastically reduced from an impact. For these reasons, numerous experimental and analytical studies have been conducted to study the dynamic response of composites subjected to impact loading. A review article [3] surveyed over 300 papers and provided a comprehensive view of the state of knowledge in this area. According to ref. [3], most of the current research effort has been focused on low velocity impact damage, specifically, the damage predictions and the evaluation and prediction of residual properties of damaged laminates. All of the research completed thus far has focused on damage in composites under impact loading in dry environments to support development of composites in aircraft structures.

As far as dynamic response of structures under water is concerned, a great deal of analytical and experimental studies have been conducted on the effect of fluid force on the natural frequencies, damping ratios and mode shapes of vibrating structures in contact with fluid. This is commonly known as the Fluid Structure Interaction (FSI) problem. FSI investigations have supported many problems in submarine signaling, offshore oil structure stability, and ship structure vibrations. Through these studies, many numerical and analytical methods have been developed in order to predict the added mass and the resulting changes in natural frequency of a structure in contact with fluid. It has been determined and widely proven that the effect of fluid surrounding a structure decreases the natural frequency of a structure due to the increase in total kinetic energy of the vibrating structure and fluid from the addition of kinetic energy of the fluid. This effect can be interpreted as an added mass to the vibrating structure in the analysis of the dynamic response. Essentially as the structure vibrates, its mass is increased by the mass of the vibrating fluid with which it is in contact, consequently decreasing its natural frequency. Studies of fluid structure interaction and the added mass effect, also known as virtual mass effect, hydrodynamic mass, and hydroelastic vibration of structures, started with Lamb [4] in 1920 who calculated the first bending mode of a submerged circular plate. In response to a problem of submarine signaling, Lamb investigated the vibrations of a thin elastic circular plate in contact with water. In his investigation he discovered that the natural frequencies for structures in contact with fluid are lower than the frequencies in air, based on the assumption that the modes shapes are virtually the same in water as in a vacuum. The resonant frequency was determined using Rayleigh's method. Powell and Roberts [5] experimentally verified Lamb's theoretical results. Lindholm et al. [6], Volcy et al. [7], Fu and Price [8] did extensive experimental study on the response of cantilever plates under various orientations, boundary conditions, geometrical shapes and levels of submergence. The above studies were mainly focused on the fundamental mode of a circular plate. Kwak and Kim [9] investigated the problem of axisymmetric vibration of circular plates. They, by employing Hankel transformation, solved the mixed boundary problem and calculated the Nondimensionalized Added Virtual Mass Incremental (NAVMI) factors for higher modes of clamped, simply supported and free plates. The NAVMI factor is the ratio of kinetic energy of water and the kinetic energy of the plate based on the assumption that the mode shape does not change under the influence of water. They also determined that for plates in contact with water on both sides, the NAVMI factor is twice the value of a plate in contact on only one side. Kwak [10] calculated the added virtual mass of rectangular plates with simply supported and clamped boundary conditions vibrating in contact with water. The Green function was used to solve the boundary value problem of the water domain. This method was combined with the Rayleigh-Ritz method. Haddara and Cao [11] presented analytical and experimental studies of the dynamic response of submerged rectangular plates. An approximate expression to calculate the modal added mass for flat rectangular plates was developed and compared to experimental results. Those FSI studies considered natural frequencies of free vibrations of wet structures.

The objective of this paper is to investigate the FSI effects on the transient dynamic responses of composite plates submerged in water and subjected to impact loading. Because

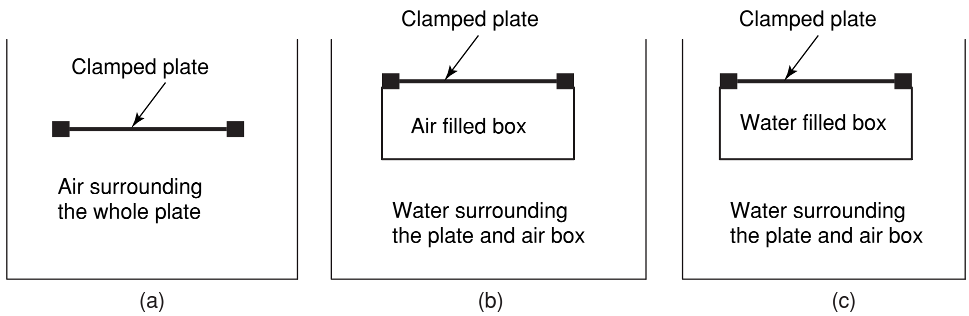


Figure 1. Three different impact conditions with composite plates held in place: (a) dry impact, (b) air-backed wet impact, (c) water-backed wet impact.

the density of the composites is close to that of the density of water, the added mass effect of the water is expected to be even more critical to the transient dynamic response of composites in water. Damage in composite plates was avoided in order to focus on their transient dynamic response with FSI.

The composites were made of carbon fiber woven fabric and vinyl ester resin using the Vacuum Assisted Resin Transfer Molding (VARTM) technique. The impact testing was conducted with a specifically developed vertical drop weight testing apparatus, and the composite samples were fitted with gages to provide real-time information on strain levels and impact forces generated during impact. The transient response of the sample included forces and strains as a function of time. Many tests were performed to verify repeatability. Phases of research included testing the samples in various environmental surroundings: dry environment for the baseline, water-backed wet, and air-backed wet environments as sketched in Fig. 1. These impact environments are described in more detail later. The dry case was completed as the baseline for comparison with the other cases in order to identify the change in response of the composite from the surrounding fluid.

2. COMPOSITE SPECIMENS AND TESTING EQUIPMENT

2.1 FABRICATION OF COMPOSITE SPECIMENS

Three carbon fiber laminate samples were constructed for this study. Each sample was fabricated from TORAY T700CF carbon fiber bidirectional weave and DERAKANE 510-A vinyl-ester matrix resin. Each plate was fabricated through the VARTM process, which consists of pulling resin through layers of carbon fibers using a vacuum. The plates consisted of eight plies oriented [0/90/0/90] at 2.38 mm nominal thickness with dimensions of 457 $\times$ 457 mm. The DERAKANE resin was mixed with three hardeners, Methyl Ethyl Ketone Peroxide (MEKP), Cobalt Napthenate (CoNAP), and N- Dimethylaniline (DMA) to achieve a nominal 60 minute curing time. The hardeners were added solely to achieve proper gel time and do not affect composite strength. All resin components were mixed based on a percent weight for a nominal cure time per manufacturer's directions at a temperature of less than 70ºF. The DERAKANE 510-A was measured by volume and the MEKP, CoNAP, and DMA were measured by weight. The VARTM apparatus consists of a glass surface, resin reservoir, vacuum pump, gauge board, and resin trap as shown in Fig. 2. The glass working surface is made of a sheet of 12 mm thick tempered glass for hardness, durability, and thermodynamic properties, as well as to promote the proper seal for the vacuum bag. The pump provides the vacuum necessary to draw the resin from the resin reservoir through the composite sample,

Figure 2. Fabrication of samples using the VARTM process.

and ensures a vacuum seal to prevent air from entering the composite sample. The gage board measures and regulates the vacuum pressure in the sample. The purpose of the resin trap is to allow air from the sample to pass freely through the gage board to the vacuum pump while simultaneously preventing the resin from contaminating these sensitive components by providing collection reservoir. After a satisfactory vacuum was established and all air leaks in the vacuum bag assembly were eliminated, inlet tubing is inserted into the resin reservoir allowing the resin to flow through the composite sample.

2.2 IMPACT TESTING APPARATUS

Impact tests were conducted using a specially designed drop weight instrumented testing system, as shown in Fig. 3. This instrumented apparatus consisted of a drop weight impactor, a load transducer, strain gages, high speed data analyzer, and an air box. The sample supporting fixture at the bottom of the drop tower was made of aluminum and facilitated square clamped conditions with a clear span of 305 × 305 mm. The composite plates were then clamped to the impactor frame using c-clamps to represent clamped boundary conditions. The transient response measurement of the sample included force and strains as a function of time.

The drop weight impactor consisted of a drop weight and an impact rod. The drop weight was supported by 4 steel guide rods, and the impact rod was supported by an aluminum frame base and a linear spring of spring constant 7508 N/m. The dimensions of the guide rods were 1.219 m high with a 6.35 mm diameter, and the dimensions of the base frame were 1.168 m high × 0.457 m wide × 0.457 m deep. The aluminum framing pieces and fasteners were designed and assembled for this research. The falling weight was guided by four small linear bearings. The impact rod was guided with two plain brushing aluminum linear bearings of 38.1 mm diameter enclosed in a casing for support. The top of the impact rod stayed above the water surface so that the drop weight would not go into the water as it hit the impact rod on its top. This fact is important not to disturb water during the impact testing

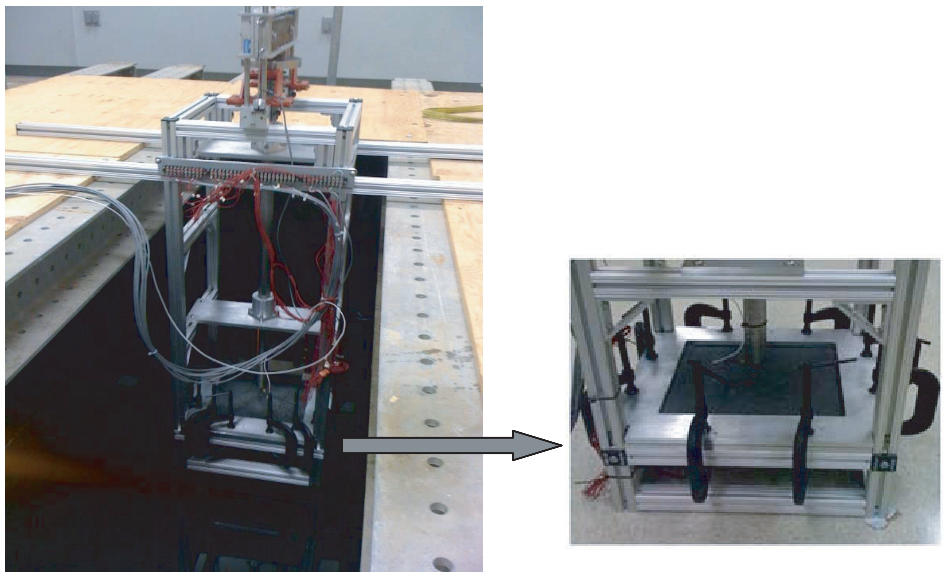


Figure 3. Drop weight instrumented testing system in anechoic water tank.

so that a composite plate interacts with still water. The other end of the impact rod, which struck a composite plate during impact, was initially very close to the specimen surface inside the water such that the disturbance of water due to the impact rod was negligible. Therefore, FSI occurred only resulting from dynamic motion of composite plates.

A trigger at the base of the falling weight was used to initiate data collection. The drop weight was kept constant at 12.0 kg. The impact rod was made of steel and had a mass of 12.7 kg. Impact energy could be varied by changing the drop height. The maximum height was 1.06 m, which could produce approximately 4.6 m/s initial velocity upon impact. The impact location was at the center of the composite sample. The selection of impact mass and height was made not to cause any damage to the composite plates so that the transient dynamic response of the plates could be focused in the study. The damage study will be conducted later and presented in a separate paper.

The load cell used was an ICP® force sensor manufactured by PCB Piezotronics, Inc. which converts force into a measurable electrical output. The load transducer was mounted on the end of the impactor rod. The gage had an impact diameter of 15.88 mm. In the case of wet testing, the gages and cable connection were coated. The strain gages were three-element 45° single-plane rosettes, model CEA-00-250UR-350, by Vishay Micro-Measurements. There were four rosette strain gages bonded to each composite sample. The gages were bonded to the underside of the laminate samples, opposite side of impact, and bonded for waterproofing. Figure 4 illustrates the orientation, location and designated x-y axis. Gage location #1 was directly at the center on the underside of the sample opposite to the impact location. Gages #3 and #4 were placed along a diagonal line of the composite plate with Gage #4 at the quarter distance of the diagonal length. Gage #2 was located close to the vertical symmetric line of the plate.

Data acquisition was carried out using an acquisition system specifically developed for this project, that consisted of a Pentium™ 4, 2.4 GHz, 512-MB RAM system, National Instruments™ simultaneous sampling multifunction DAQ, and five Vishay™ 2120 multi-channel strain signal conditioners. The system had 16 bit analog-to-digital conversion resolution and was capable of reading a total of 16 channels at a throughput rate of up to

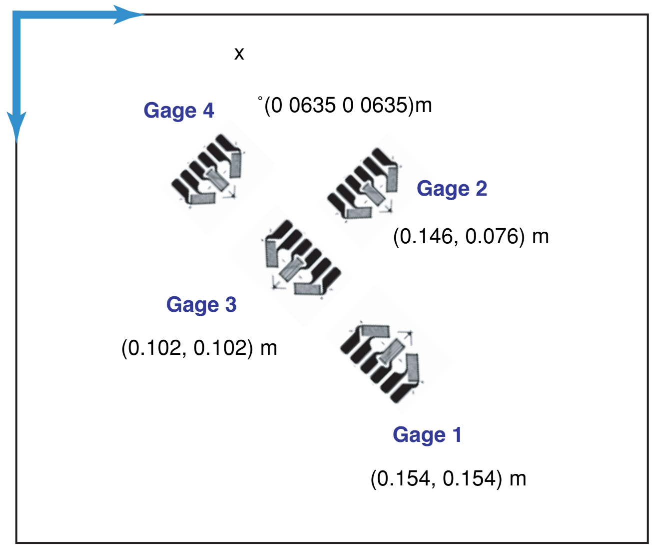


Figure 4. Strain gage rosette locations opposite the side of impact. (The plate size is 0.305 $m \times$ 0.3045 m).

250 kS/s per channel, which was appropriate for the rate of testing used in this study. The data-acquisition process was controlled using the NI-DAQmx driver software and LabVIEW™ interactive data-logging software that was specifically formatted for this research. A trigger located at the top of the impact rod was used to initiate data acquisition. Strain readings from four signal conditioners were multiplexed in order to accommodate all strain gages within the available number of channels. Errors due to instrumentation noise did not appear to cause problems in the data capture so no filtering was used.

An air box was constructed to facilitate testing for air-backed wet environments. The box was made of 12.7 mm thick plexi-glass with dimensions 330 mm wide × 330 mm long × 127 mm deep. This box was then secured to the bottom aluminum support plate for the composite sample using 8 c-clamps of dimensions 76 mm jaw × 60 mm throat, and sealed with putty tape to prevent water leakage. The box completely covered the sample so that the bottom side of the plate was not exposed to water. A 19 mm diameter hole was cut out from the side to feed the wiring from the strain gages to the data analyzer, which was filled with putty to prevent water leakage during testing.

An anechoic water tank used for underwater surroundings testing was measured 2.75 m wide × 2.75 m long × 2.75 m deep. The anechoic tank was used to minimize the influence of the wave reflection from the boundary walls. The tank was filled with fresh water. A standing platform was constructed across the top of the tank made with aluminum I-beams and plywood, leaving a 0.635 m × 0.914 m square opening for suspension of the drop weight impactor.

3. IMPACT TESTING

Three different impact cases were studied in order to examine the effects of FSI on composite structures under dynamic loading. These cases were shown in Fig. 1. First, the dry impact was conducted as the baseline. For the dry impact test, the composite plates were impacted without having any contact with the water. This is shown in Fig. 1(a). Subsequent wet impact tests were undertaken for the same composite plates. In order to avoid any moisture effect on the composite materials, the wet impact testing was performed as soon as the composite plates were submerged into the anechoic water tank. Furthermore, once the wet impact testing was completed, dry impact of the plate was conducted immediately following the wet impact. The responses of the dry impact tests before and after the wet impact testing were compared. Their results were consistent. By doing so, it could be verified that the composite plates did not absorb any moisture to affect their material properties.

Two different wet impact conditions were considered. The first case has an air-containing rigid box attached to the bottom of the composite plates. The box was completely sealed so that no water penetrated into the box when the composite plate with the attached box was submerged into the water of the anechoic water tank. Then, impact loading was applied to the composite plate submerged in water. This is called the air-backed wet impact and is shown in Fig. 1(b). The air-backed composite plate is only in contact with water at the top side where the impact occurs. The other wet impact case was very similar to the previous one except that the air-box was no longer sealed so that water filled the box when the plate and the box were submerged into the water tank. This is sketched in Fig. 1(c) and called the water-backed wet impact. The water-backed plate is exposed to water on both sides in this case. The same impact conditions, i.e. the same drop weight and height, were applied to the three impact loading cases. The wet impact responses were compared to the dry impact data in order to evaluate the FSI effects.

4. EXPERIMENTAL RESULTS AND DISCUSSION

Impact testing was conducted for both dry and wet composite plates. The mass of the free falling object was 12 *Kg* which was dropped from the height 1.07 *m*. In order to confirm the repeatability of the impact test data, every test condition was repeated several times for the same composite plate. The measured force and strain data were very close one another. This fact confirmed not only repeatability but also no damage in the specimen. If damage occurred and accumulated in the composite plate, repeated testing would show different results with the damage. Figure 5 compares the two force data curves under the same dry impact condition. Other force data, which

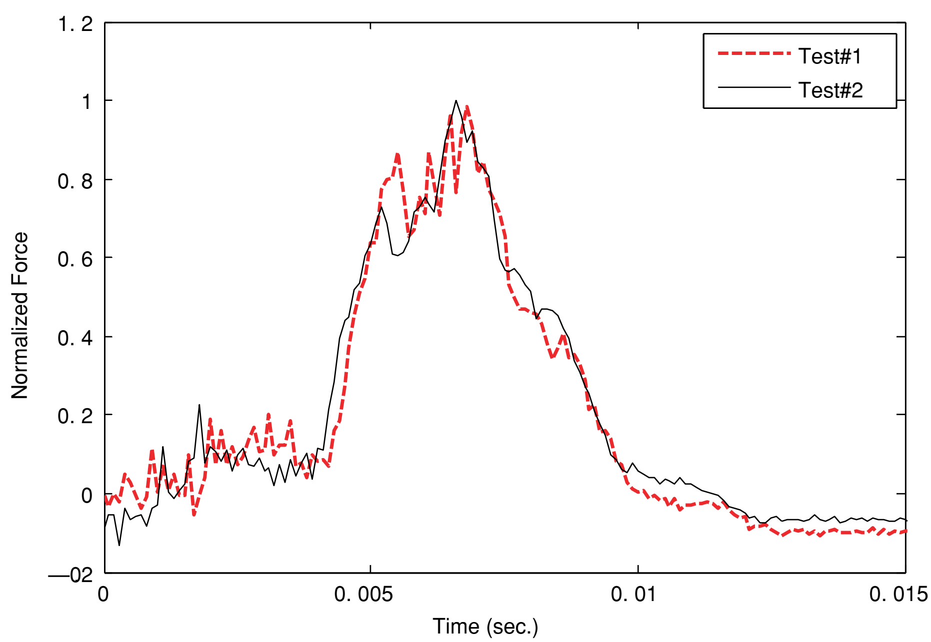


Figure 5. Comparison of dry impact forces under the same impact condition.

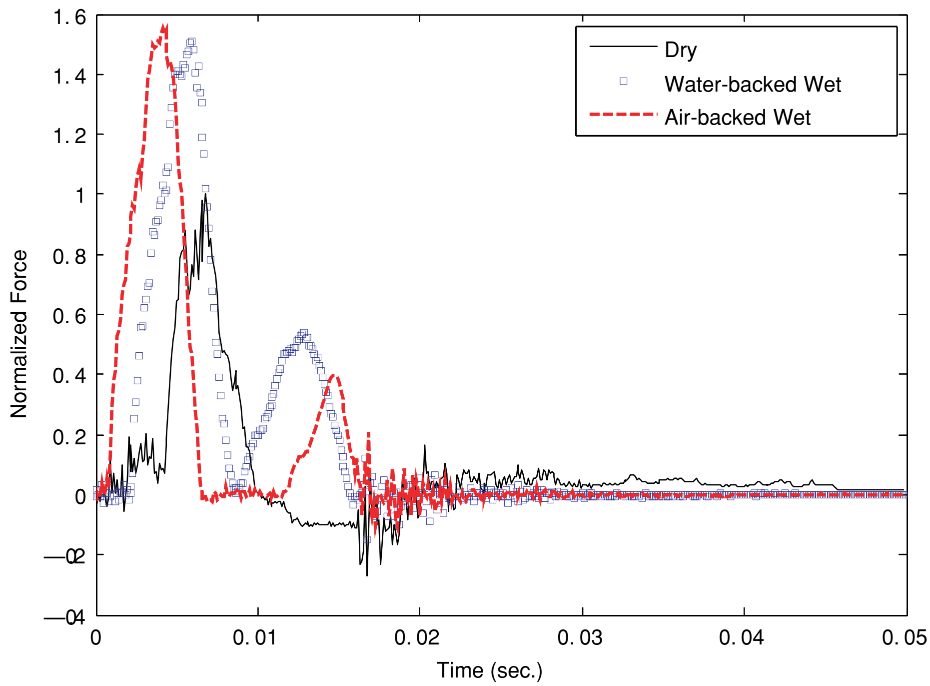


Figure 6. Comparison of impact forces among dry, water-backed and air-backed wet impact cases.

were not plotted here to avoid crowding, are very close to the graphs shown in the figure. In all figures unless otherwise mentioned, the force or strain plots were normalized in terms of the dry impact data so that the effects of FSI could be better represented in the plots.

The impact force is compared in Fig. 6 for the dry impact as well as the water-backed and air-backed wet impact cases. As shown in this figure, the air-backed and water-backed wet impacts yielded 55% and 50% greater impact force than the dry case, respectively. The larger wet impact forces were caused by the hydrodynamic added mass effect. Because the density of the composite plate is only 60% greater than that of water, the plate with the added mass moves with a much slower velocity. As the plate moves more slowly, the contact force between the impactor and the plate becomes larger, which is recorded to the force gage. In order to support this argument, a series of finite element analyses were conducted. In the numerical study, the same composite plate was subjected to an impact loading condition with the same impactor mass and velocity as in the air experiment. However, the impact rod and the spring were not considered in the model in order to simplify the numerical model. The peak impact force was obtained from the analysis. Then, the mass density of the composite plate was increased uniformly over the plate by 50% or 100%, respectively, while all other conditions including the plate stiffness and impact condition remained the same. The increase of the mass density was to represent the added mass effect with wet impact. The real added mass would be non-uniform over the plate. However, since the actual distribution was not known a priori, a uniform distribution of added mass was considered in the finite element analysis. The analyses showed that as the composite mass increased by 50% or 100%, the peak impact force increased to 20% or 30% when compared to that with the original mass. Because the wet structure certainly does not have a uniform added mass effect, the actual increase of the peak force in the experiment was different from the finite element analysis results. However, the analyses support the experimental results qualitatively as stated previously.

Both wet impact forces have steep monotonic increases to their peak values just after the impactor hit the plates while the dry impact force shows an initial low plateau before it moves steeply to its peak value. Because the impactor was not held after the initial impact, it rebounded and landed again. As a result, both wet impacts showed secondary peak forces. However, the dry impact did not have the secondary contact. In particular, the air-backed wet impact case gave a quite a delay between the initial and secondary impact forces while the water-backed wet impact case showed the secondary force occurring just after the initial one.

The reason that the air impact test did not produce the secondary impact was due to the spring in the impact test machine. The spring supports the impact rod, which will hit the composite plate, a small distance above the plate initially before the impact loading. As the impact weight drops and hits the impact rod, the latter moves down with compression of the spring to hit the composite plate. For the air impact case, the initial impact force was lower than that for wet impact cases. Therefore, the redounding and landing force from the air impact was not large enough to overcome the spring force so that the rod could not hit the plate again.

Because strain gage location #1 lies directly underneath the impact site of the composite plate, the strains contain many higher frequency components compared to strains at other locations. A comparison of Fig. 6 to Fig. 7 shows that the peak values of impact forces and strains at position #1 occur simultaneously. Strains under wet impacts are more than double the dry impact strain at location #1. This ratio of peak strains between the wet and dry impacts is even greater than that of the impact forces.

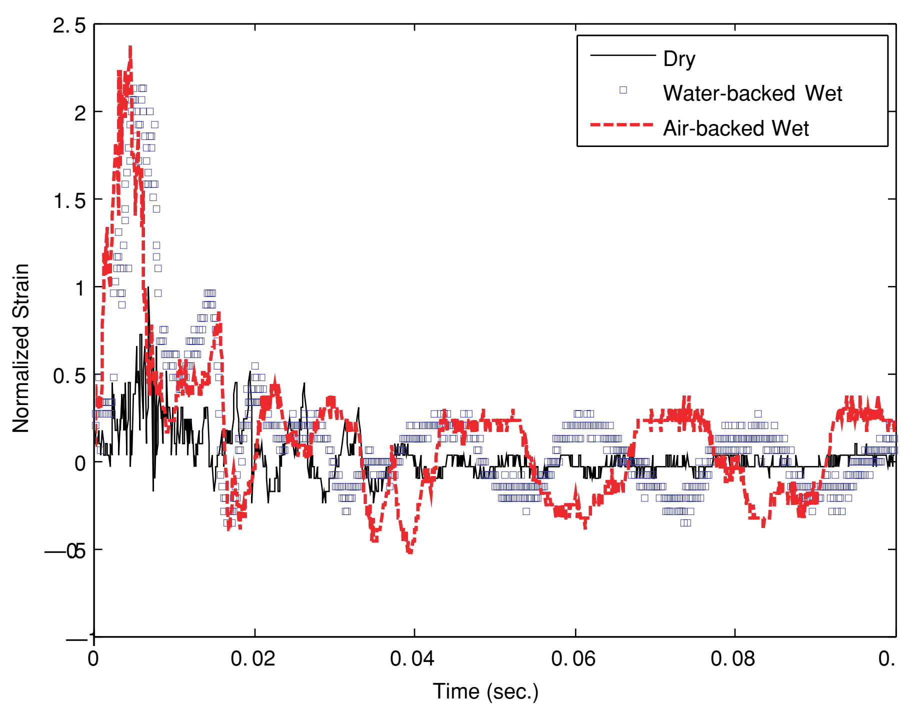
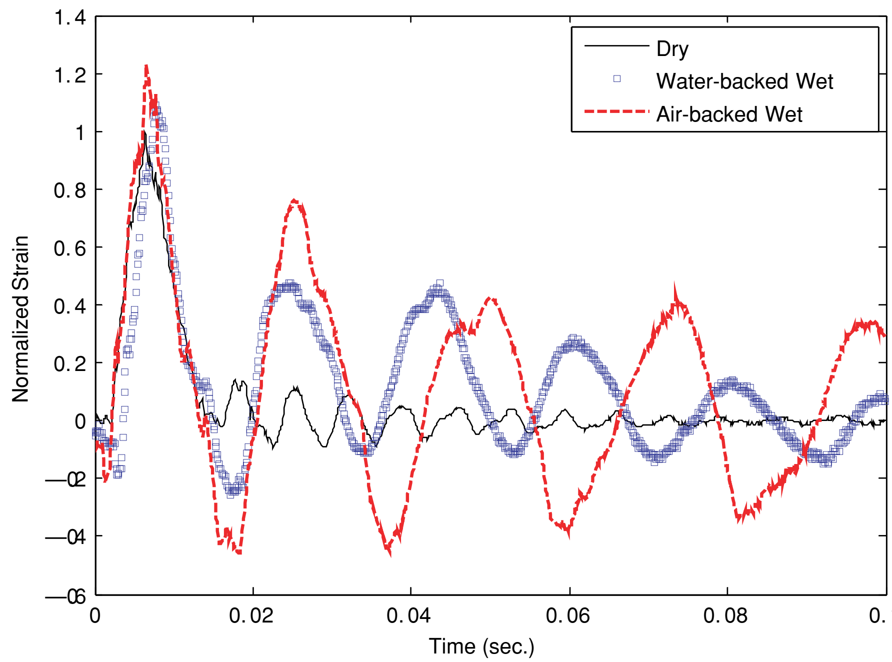
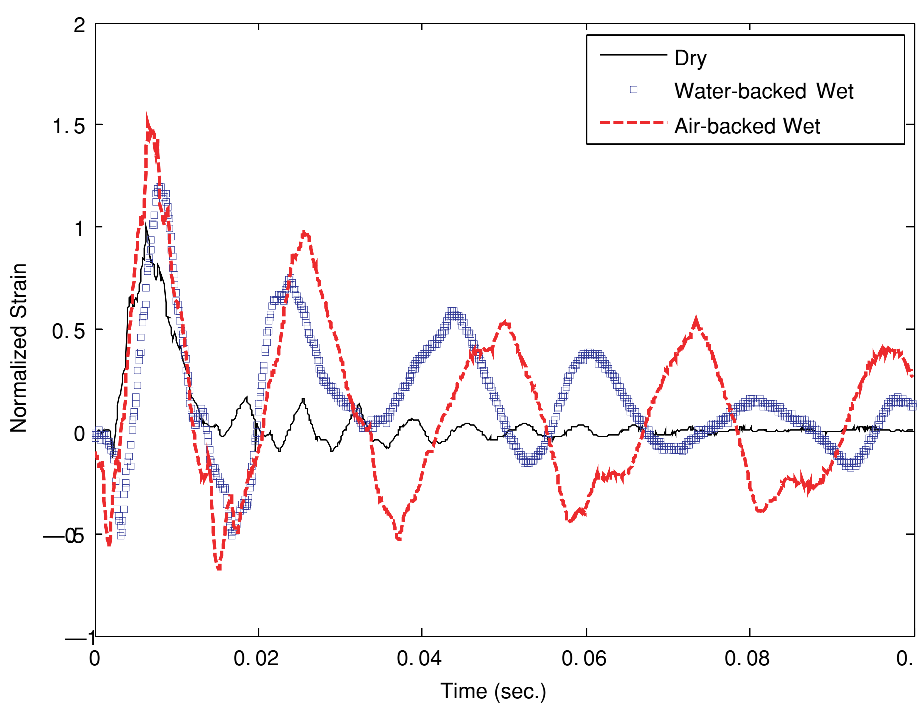


Figure 7. Comparison of strains along *x*-axis at position #1 among dry, water-backed and air-backed wet impact cases.



(a) Strain along x-axis



(b) Strain along y-axis

Figure 8. Comparison of strains at position #2 among dry, water-backed and air-backed wet impact cases.

The differences between the wet and dry impact strains were smaller at gage location #2 than those at location #1. The strains under the air-backed wet impact were 20% and 50% greater than the dry impact strains along the *x*-axis and *y*-axis, respectively, as shown in Fig. 8. On the other hand, the water-backed impact resulted in 10% and 20% greater strains in the *x*-axis and *y*-axis, respectively, than the dry impact. The gage location #2 is closer to the clamped boundary in the *y*-axis direction. Thus, this suggests that the clamped boundary resulted in a greater FSI effect on the composite plate. Both wet impact strains showed initial compressive strains before much larger tensile strains. Furthermore, these strain measurements show a clear difference among the response frequencies due to the added mass effect. Comparing the wet impact responses to the dry impact response, the response frequencies under the wet impacts are less than a half of the dry impact response frequency. Such a drastic reduction is caused by the light composite structure which is only about 1.6 times as heavy as the water in terms of density. The response frequency is higher for the water-backed wet impact case than for the air-backed wet impact case by approximately 20%. This is an interesting result because the water-backed wet structure was expected to have a greater added mass effect with a lower response frequency. However, as was expected, the decay of the strain peak values was greater for the water-backed wet impact case than the air-backed case. For example, the average damping ratio was 0.053, 0.062, and 0.11 for the dry, air-backed, and water-backed impact cases, respectively. This means the damping effect is greatest for the water-backed case.

Calculation of the Added Virtual Mass Incremental Factor (AVMIF), β from Eq. (1) given below, yields approximately 6.5 and 11.5 for the water-backed and air-backed wet composite plates, respectively.

$$\frac{\omega_w}{\omega_d} = \frac{1}{\sqrt{1+\beta}} \tag{1}$$

where ω is the frequency and subscripts w and d denote the wet and dry cases, respectively. AVMIF represents the ratio of the kinetic energy of the water to that of the composite plate. The AVMIF for steel submerged in water ranges from 1.4 to 2.4 depending on the boundary conditions [8, 11]. Comparison of AVMIF between the composite and steel shows clearly a much larger effect of FSI on the composite than steel.

As seen in Fig. 9, the strain gage readings at gage location #3 have similar response characteristics as observed in the gage at location #2. Both air-backed and water-backed wet impacts resulted in 30% greater strains in *x*-axis than the dry impact. Because the gage location #3 is on the diagonal direction, strains in the *y*-axis were very close to those in the *x*-axis.

The effect of FSI is very significant for the strain gage reading at location #4, as shown in Fig. 10, and the strain readings were much less harmonic with more constraint effects from the clamped boundaries of the plate. First of all, both wet impact cases resulted in very large initial compressive strains compared to the dry impact case. The initial compressive strains were due to the clamped boundary conditions. After following the initial large compression, the air-backed wet impact case shows another large tensile strain while the water-backed case has a modest magnitude of tensile strain. The water on the backside of the water-backed composite plate seems to prevent further tensile strain at this gage location. The magnitudes of the strains at the gage location #4 are 4 to 5 times higher for the wet impact cases.

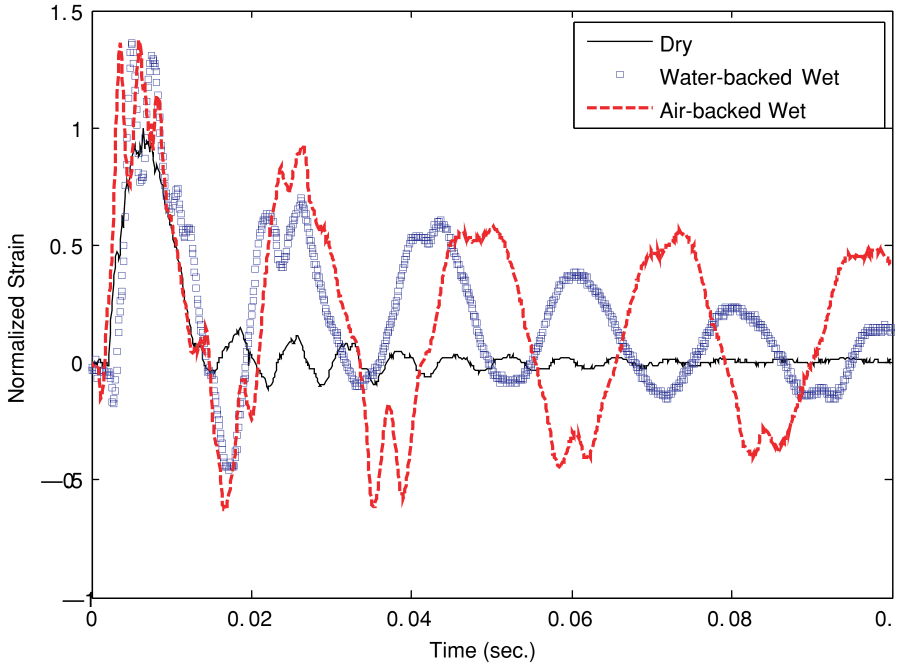


Figure 9. Comparison of *x*-strains at position #3 among dry, water-backed and air-backed wet impact cases

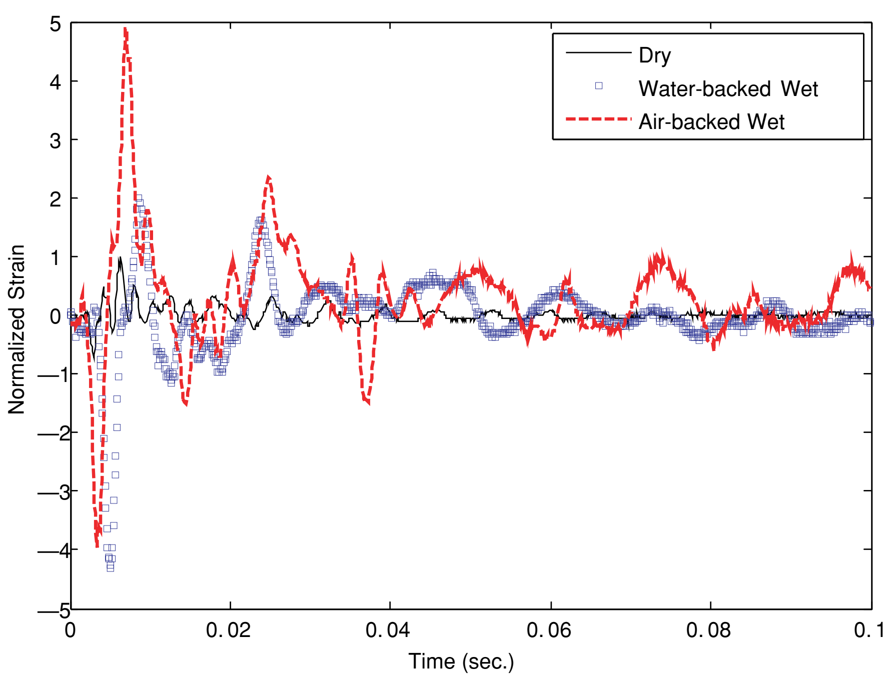


Figure 10. Comparison of *x*-strains at position #4 among dry, water-backed and air-backed wet impact cases.

5. CONCLUSIONS

An experimental set-up was designed and fabricated for impact testing on composite plates submerged in water. In order to investigate the transient dynamic response of composite plates with FSI effects under impact loading; three impact conditions, dry, air-backed wet, and water-backed wet impact conditions were considered, respectively. In order to focus on the FSI effects on the transient dynamic responses, impact loading was controlled not to cause any damage to the composite plates. Since the composite material has a very comparable density to water, the FSI effects were very significant on the impact force and transient responses of the plates. Due to the added mass effect of water, the impact force was much greater for the wet impact cases than in the dry impact. Similarly, wet impact produced much greater transient strains on the composite plates. As a result, the wet impact was more detrimental to the structure than the dry impact. However, the increase of magnitude of transient strain responses resulting from the FSI with wet impacts varied significantly depending on the location of the composite plate because the added mass effect was not uniform over the plate. The location near to the clamped boundary corner had generally a greater FSI effect on the transient strain response. This suggests that very careful evaluation is necessary for design and analysis of composite structures for underwater application subjected to transient dynamic loading. Finally, comparison of AVMIF between the composite and steel shows approximately 3 to 5 time greater values for composites than steel, which indicates a much greater effect of FSI on the composite than steel.

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