Uncertainty analysis for dynamic properties of MEMS resonator supported by fuzzy arithmetics

Adam Martowicz[1], Irina Stanciu[2] and Tadeusz Uhl[1]
[1]Department of Robotics and Mechatronics
AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
[2]Laboratory of Modelling and Simulation
National Institute for R&D in Microtechnologies (IMT-Bucharest)
PO Box 38-160, Bucharest 72225, Romania
adam.martowicz@agh.edu.pl, irina.stanciu@imt.ro, tuhl@agh.edu.pl

ABSTRACT

In the paper the application of uncertainty analysis performed for microelectromechanical resonator is presented. Main objective of undertaken analysis is to assess the propagation of considered uncertainties in the variation of chosen dynamic characteristics of Finite Element model of microresonator. Many different model parameters have been assumed to be uncertain: geometry and material properties. Apart from total uncertainty propagation, sensitivity analysis has been carried out to study separate influences of all input uncertain characteristics. Uncertainty analysis has been performed by means of fuzzy arithmetics in which alpha-cut strategy has been applied to assemble output fuzzy number. Monte Carlo Simulation and Genetic Algorithms have been employed to calculate intervals connected with each alpha-cut of searched fuzzy number. Elaborated model of microresonator has taken into account in a simplified way the presence of surrounding air and constant electrostatic field.

1. INTRODUCTION

Launching a new product into market comes after its virtual and real prototyping is accomplished [1]. Investigation of its performance is related to a number of dynamic and static properties. During virtual prototyping they are studied via deterministic and nondeterministic simulations performed for numerical models elaborated e.g. with Finite Element Method (FEM). As far as nondeterministic analyses are of engineer's concern an uncertainty analysis is used to study the propagation of assumed parameters variabilities in chosen characteristics. Uncertainty analysis allows for the introduction in the analysis the fact that product's properties in reality characterize uncertainty. It appears as subsequent items of the same mechanical structure do not have the same geometrical dimensions, are not made with totally same material and finally do not operate in the same environmental conditions. Uncertainties always appear and their introduction into carried out analyses enables to yield better results and helps to predict scatter of product's properties while operation.

As manufacturing processes of microelectromechanical systems (MEMS) [2] do not guarantee infinite repeatability of their characteristics, technological and material uncertainties have to be taken into account to make results of computer simulations as close to reality as possible [3,4]. Performed uncertainty analyses enable more realistic ranges of variation of analyzed characteristics e.g. in terms of present geometry imperfections and changes of material properties of studied device. For uncertainty analysis, microresonator has been chosen as one of most commonly manufactured and exploited MEMS [5,6,7]. It is used e.g. in filters, accelerometers, clock and frequency reference applications. Microresonator with two comb drives and area-efficient folded flexures is studied. In performed simulations multiphysics approach has been considered and phenomena of air damping and electrostatic field have been modeled. Selected operational microresonator resonance frequency has been studied because of its crucial influence on device performance. In present work, uncertainty analysis has been performed by means of fuzzy numbers theory combined with alpha-cut strategy. Output fuzzy numbers have been assembled with intervals obtained by applications of Monte Carlo Simulation (MCS), supported by Latin hypercube procedure of samples generation, and Genetic Algorithms (GA). Additionally, a sensitivity analysis has been carried out to order given uncertainties according to their separate influences on analyzed frequency of vibration. All considered uncertain parameters have been modeled by fuzzy numbers with triangular shapes of membership functions. The set of uncertainties covers a number of geometry characteristics: length, width, location of comb drive fingers, length and width of flexure beams, thickness and angle of chamfer of the device. Moreover, some material properties have also been assumed to vary: Young's modulus, Poisson's ratio and density of polisilicon.

2. UNCERTAINTY ANALYSIS, SENSITIVITY ANALYSIS

The objective of uncertainty analysis is to study the overall propagation of identified uncertainties, called as input parameters of analysis, in interesting characteristics that are, in turn, output parameters of analysis. In engineering practice it seems to be important issue as, in reality, variation of crucial product's properties can be observed and appears due to variation of technological, material and operational characteristics. Successive products differ one from another and this fact motivates applications of uncertainty analysis as effective complement to commonly used deterministic analyses. In Figure 1, exemplary workflow in uncertainty analysis is presented. It takes into account both numerical and experimental part of analysis.

All activities involved in the uncertainty analysis can be divided in 3 parts. They, in sequence, deal with the following tasks:

a) establishing the set of uncertain parameters that are taken into uncertainty analysis;
   Identification of uncertainty deals with the elaboration of the set of all uncertain parameters that are taken in uncertainty analysis. The set should contain parameters of which real variability can be measured or assumed accordingly to expert's knowledge. Moreover, as input parameters of the study, properties which can be controlled during manufacturing process should be taken into account since this approach potentially creates possibility of improving product performance via reduction of variability of output characteristics. The more uncertainties are taken into account the more realistic results of analysis should be. However, it should be also highlighted strong recommendation of the usage of sensitivity analysis when dealing with great number of uncertain parameters. It happens that a number of them can be neglected in assessment of uncertainty propagation as they do not have any significant influence on interesting output characteristics. The conclusion is that the introduction of many different parameters is expected but study of their sensitivity is required to keep only important ones. Identification of uncertainties should deal with all stages of product's life: design, manufacturing process and operation [8,9]. During design uncertainties can represent: multiplicity of solution concepts, topology and number of structural elements, incompleteness of information on material properties as well as variety of

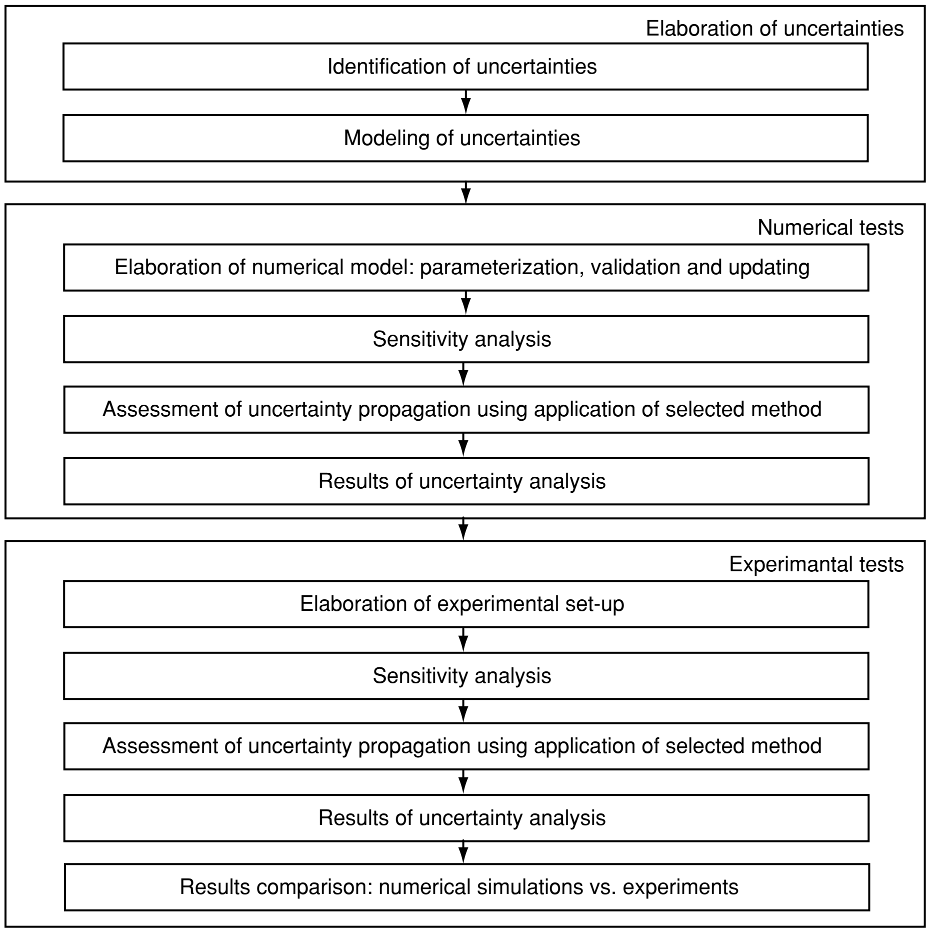

Figure 1 Exemplary workflow of uncertainty analysis.

techniques that may be used to model physical phenomena. Uncertainties which appears in manufacturing process are connected with variation of quality of used processes, tools, geometry tolerances, quality of joints etc. Finally, at operation uncertainties describe changes of the environmental and loading conditions as well as changes of properties due to aging etc.

Modeling of uncertainties deals with the choice of methods that are used to represent variabilities of input parameters [8,10,11]. On the basis of available measurements and assumed data the process of uncertainty modeling is performed employing both probabilistic and possibilistic techniques [10,11]. Depending on the form of representation of variability and considering accessible statistics, random numbers, random fields, intervals and fuzzy numbers can be applied. Some help in choice of method of uncertainty modeling arises from the distinction between reducible and irreducible uncertainties [8,9,10]. Reducible uncertainty, also called as epistemic uncertainty or subjective uncertainty, means potential deficiency of knowledge on input parameters. Present lack of knowledge is gradually eliminated as required information is gathered. In case of reducible uncertainty it seems to be understandable that possibilistic techniques are preferably used to model this kind of uncertainty. In this case, usually no statistic data is available. Irreducible uncertainty, called also as aleatory uncertainty or just simply as variability, represent immanent, inevitable uncertainty, substantially connected with physics of modeled mechanical system. It is not possible to eliminate totally this kind of uncertainty. Most known examples of irreducible uncertainties are manufacturing tolerances or changes of environmental conditions. Irreducible uncertainties can usually be characterized by statistic parameters such as probability density functions (PDF) and statistical moments and therefore probabilistic methods can be applied to elaborate input domain of uncertainty analysis.

b) numerical analyses;
First, numerical deterministic model is required to evaluate value of interesting characteristics for given combinations of values of input uncertain parameters. In general however, it would be expected to have nondeterministic model that on the base of given input uncertainties could directly estimate variability of output characteristics. The task to obtain that kind of model is not a trivial one especially in the context of building of FE model. This fact motivates alternative solution which is based on sequential repetition of numerical simulation of deterministic model in order to assemble nondeterministic final form of results. To summarize, having elaborated deterministic model it is also possible to find variability of output characteristics by the use of deterministic analyses performed in simulation loop until all required combinations of values of input uncertain parameters are checked accordingly to the chosen method of assessment of uncertainty propagation.

On the basis on experimental measurements of physical prototypes a parameterized model is validated, i.e. correct structure of model is found, and updated, i.e. correct values of model properties are found.

Prepared model is ready to be used in sensitivity analysis. The objective of sensitivity analysis is to find separated influences of input uncertain parameters on interesting output characteristics. The results of sensitivity analysis can be used to eliminate noninfluential parameters from input domain. This possibility is especially important for applications of possibilistic methods in which exact number of model realizations is needed and this number strongly depends on the number of input parameters.

Next step of analysis deals with the assessment of uncertainty propagation. Depending on both the way how input uncertainties are modeled and required form of results, a method to assess interesting variability is chosen. All methods can be divided in 2 groups: probabilistic methods and possibilistic ones [10,11]. Probabilistic methods use e.g. random variables or random fields. They can be used to find statistical parameters such us means, standard deviations or higher-order statistical moments. It is also possible to find histograms of output quantity. MCS is most commonly applicable probabilistic method, used both in crude version and with more sophisticated methods of sampling the input parameter domain. The latter approach allows for the reduction of the required number of samples and better their locations in input parameter space. Possibilistic methods, in turn, are used when there are limitations concerning knowledge on the PDF and statistics of the input parameters. The examples of possibilistic methods are: interval analysis [12], vertex method [10], fuzzy sets theory with Zadeh's extension principle [13], transformation method and its modifications [14,15].

c) experimental analyses;
In case when uncertainty analysis based on real measurements is possible, i.e. when a

series of physical prototypes can be produced, it is possible to verify both the results obtained in uncertainty analysis supported by numerical simulations and quality of methods used to assess the uncertainty propagation. Elaborated experimental set-up should allow for possibly great number of tested product items and easy measurements of input and output characteristics.

As stated in the description of numerical case, having information on variabilities of input parameters, the methodology to find overall uncertainty propagation is chosen and then applied. Successive items of products are produced and all necessary scatters and statistics of input uncertainties and output characteristics are gathered. On the basis of measurements final form of results representation is then assembled.

In presented paper only numerical part of uncertainty analysis is described. Amongst all uncertainties which are present in MEMS only geometrical and material ones have been introduced and their propagation in FE model of microresonator has been studied. Possible defects and faults have not been considered in the work as they have been treated as catastrophic changes of device properties that disable its proper operation [16]. In present study theory of fuzzy numbers with alpha-cut strategy have been applied to find uncertainty propagation.

3. FE MODEL OF MEMS RESONATOR, STUDIED CHARACTERISTICS

FEM has been used to model MEMS resonator as this method seems to be most commonly applied to represent properties of real mechanical structures in numerical manner. The variability of chosen operational resonance frequency has been chosen as the object of the study. In Figure 2 the elaborated model and chosen operational mode are presented. Nominal value of studied natural frequency equals 136,962Hz.

Presented model is built with movable shuttle mass suspended over the substrate using 2 area-efficient folded flexures. MEMS resonator is equipped with 2 comb drives that allow for activating the device and measurement the displacement along longitudinal axis x. The microresonator can be externally actuated to vibrate with operational frequency by the use of mounted comb drives. The overall dimensions of the movable part are 288μm and 186μm. The thickness of microresonator equals 3μm. The gap between moving part and substrate is 3μm. Model mesh consists of 634 finite elements (FE) connected in 2072 nodes. Figure 3 presents the description of structural parts of the model.

Multiphysics approach is considered in the model. Air damping and electrostatic field are of concern and their presence is introduced in a simplified way [2]. Additional elastic and damping discrete elements have been introduced to model mentioned phenomena. All 6 locations (3 pairs at top and bottom respectively within horizontal surfaces in plane x-y) of used 2 springs and 6 dampers are presented in Figure 4. Free element connections are fixed to the substrate.

Introduced mechanical elements represent the influences of surrounding air (dampers) and electrostatic field (springs) considered only for axis x, along which the motion of movable part can be observed at resonance operation. The influences that appear for other normal modes are intentionally skipped for the simplicity of the calculations. Introduced elements have been modeled using PBUSH/CBUSH elements in MSC/ Nastran software.

Resultant stiffness and damping coefficients are calculated on the basis of the geometry characteristics of current realization of FE model and then used as properties of introduced discrete elements. This approach is treated as an alternative to the formulation of coupled

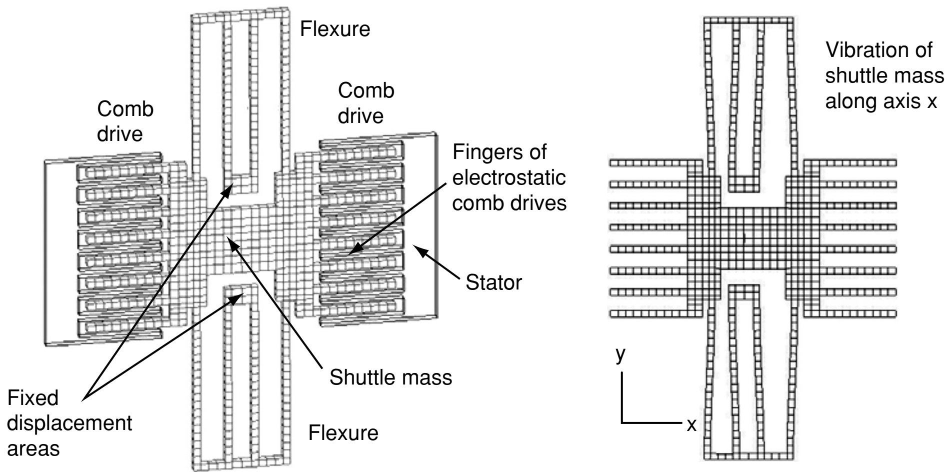

Figure 2 FE model of MEMS resonator and operational normal mode.

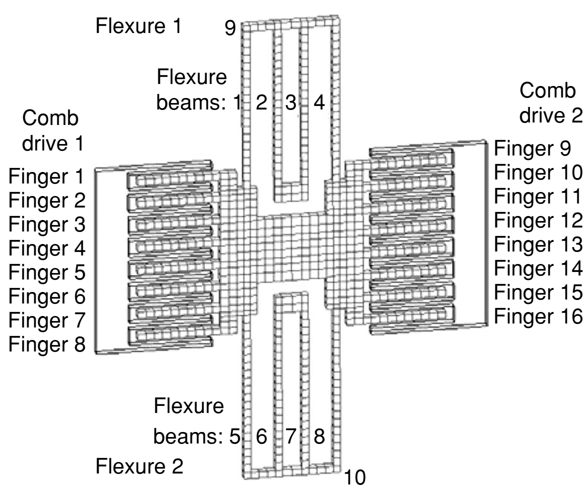

Figure 3 Structural components of the model.

problems [2] and enables for significant time-savings. For presented case time needed to perform whole calculation loop using simplified way do not extend 1 minute whereas in case of solution of coupled problem the same results can be achieved after few minutes. At this point, it should be mentioned that in case when only one normal mode of presented model is studied, there is a possibility of farther simplification and shortening the computational time. As in modal analysis, each normal mode can be substituted by a one-degree-of-freedom system, i.e. simple oscillator. Properties of this oscillator, i.e. resultant mass, stiffness and damping coefficient, could be calculated by the use of empirical relationships elaborated on the basis of results of numerical simulations or measurements of real characteristics. However, the quality of used empirical equations could be problematic, especially in the context of great number of model parameters with many geometry characteristics. In fact, the

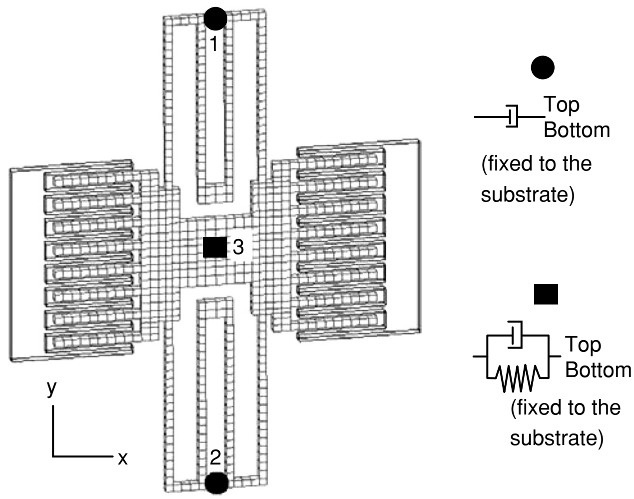

Figure 4 Discrete elements used to represent air damping and electrostatic field.

procedure of mesh morphing of FE model as well as its simulation take some time but also enable to consider all changes in geometry easily and accurately.

3.1. AIR DAMPING IN FE MODEL

In the model 4 types of phenomena related to air damping have been considered:

a) slide-film damping force;
It appears when parallel movement of an object with respect to surrounding air is observed. Viscous damping is then interpreted as coming from force arising in presence of relative displacement of neighboring layers of fluid that is in contact with object immersed in it. During calculation area of longitudinal cross-section of moving object is important.

b) drag force;
It is observed when there is a movement of an object through the air. I this case its area of transversal cross-section is important.

c) squeeze-film damping force;
It appears in case when an object moves against other element which is very close located. This kind of damping expresses the phenomenon of under- or overpressure that occurs in case of small gaps.

d) acoustic energy dissipation;
It represents energy dissipation due to activating acoustic waves spreading through air. Each object vibrating in the fluid is a source of acoustic energy.

All damping coefficients are calculated separately for each damper depending on the part of FE model for which represent phenomenon appears.

As far as slide-film damping is of concern, 2 cases have been considered. First one assumes Couette flow which means the situation when gap dimension between movable part and surrounding substrate is lower than effective decay distance δ formulated by the following expression [17]:

$$\delta = \sqrt{2\mu/\rho\omega} \tag{1}$$

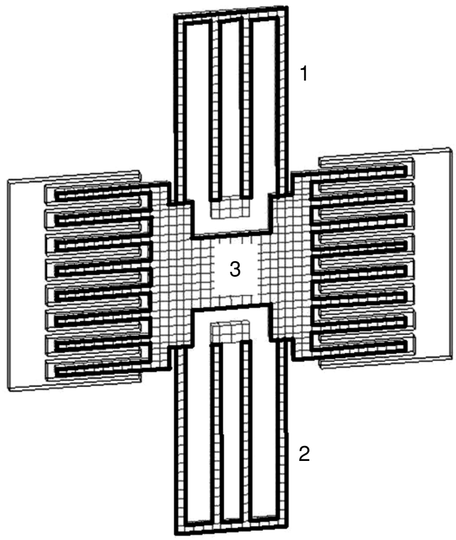

Figure 5 Slide-film damping phenomenon - considered horizontal areas of moving parts.

where: μ is kinematic viscosity of fluid ($1.81 \cdot 10^{-5}$Pa · s for air), ρ is fluid density (1.2kg/m^3 for air) and ω is circular frequency of vibration. For elaborated model parameter δ equals 5.7μm. Considering horizontal surfaces of model (in plane x-y), Couette flow has been assumed under the shuttle mass since the distance between it and anchor is 3μm. Second kind of flow which is Stokes flow should be taken into account when the distance gap between objects in fluid is grater than δ. This case appears over modeled MEMS resonator. Similarly, depending on distances between movable resonator parts and substrate, also for all sidewalls (vertical surfaces of model) both Couette and Stokes flows have been introduced to represent correctly the phenomenon of slide-film damping.

Figure 5 and 6 present all areas that have been considered during calculation damping coefficients expressing phenomenon of slide-film damping. Figure 5 relates to horizontal surfaces of moving parts whereas Figure 6 defines vertical ones. Used numbering of areas marks related pairs of dampers.

To calculate related damping coefficients c the following formulas have been used [17]. In case of Couette flow slide-film damping force F can be calculated with the expression:

$$F = \mu \frac{A}{d} \dot{x} = c\dot{x} \tag{2}$$

whereas for Stokes flow F can be found with the formula:

$$F = \mu \frac{A}{\delta} \dot{x} = c\dot{x} \tag{3}$$

where A is area of longitudinal cross-section of moving part of microresonator, d is thickness of air film, $\dot{x}$ is velocity of shuttle mass along axis x.

All areas considered for the phenomena of drag force, squeeze-film damping and energy dissipation via activating acoustic waves are presented in Figure 7.

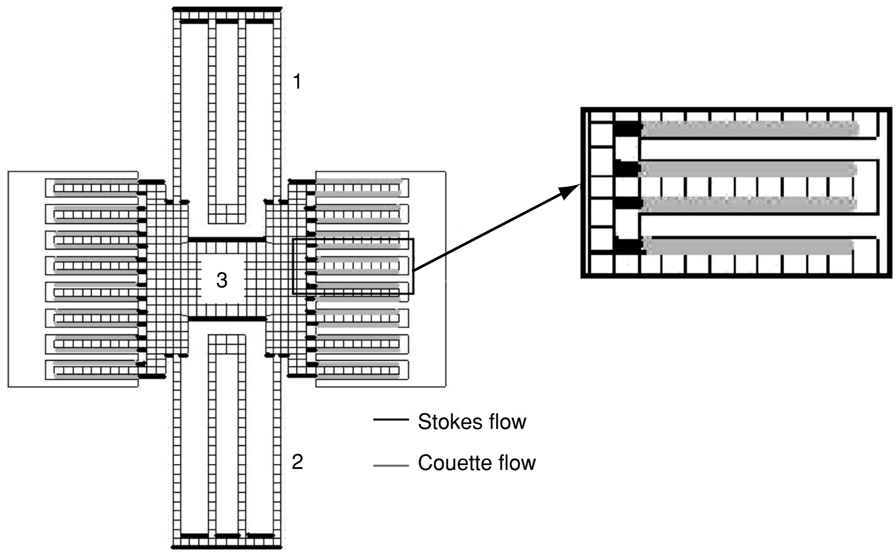

Figure 6 Slide slide-film damping phenomenon - considered vertical areas of moving parts.

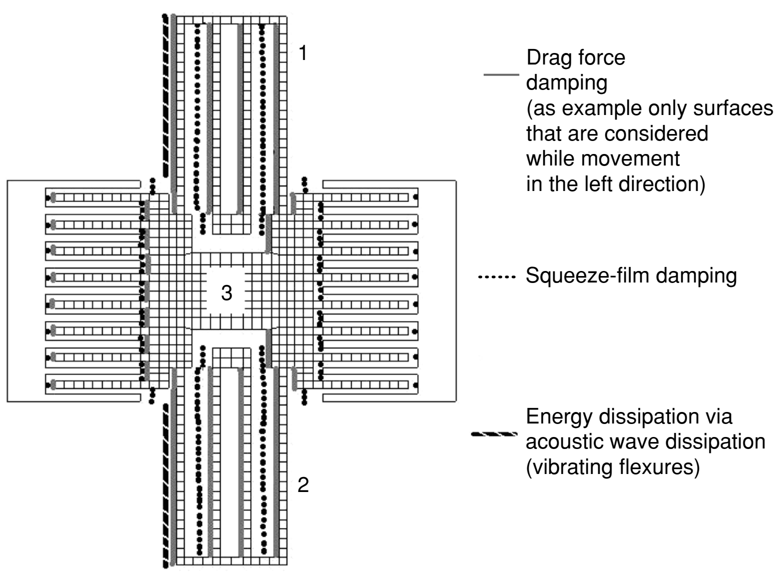

Figure 7 Drag force, squeeze-film damping force and acoustic wave dissipation phenomena - considered areas of sidewall surfaces of moving parts.

Damping coefficients which have been used to represent presence of drag force can be calculated with the expression [17]:

$$F = \frac{32}{3}\mu r\dot{x} = c\dot{x} \tag{4}$$

where r stands for characteristic dimension. It represents a half of grater dimension in case of rectangular shape of object moving through air.

In case of squeeze-film damping force it is calculated with following expression:

$$F = \mu\frac{LB^3}{d^3}\beta\left(\frac{B}{L}\right)\dot{x} = c\dot{x} \tag{5}$$

where L, B are dimensions of rectangular shape (assuming that $L > B$) and factor $\beta\left(\frac{B}{L}\right)$ is found by the equation [17]:

$$\beta\left(\frac{B}{L}\right) = \left\{1 - \frac{192}{\pi^5}\left(\frac{B}{L}\right)\sum_{n=1,3,5}^{\infty}\frac{1}{n^5}th\left(\frac{n\pi L}{2B}\right)\right\} \tag{6}$$

Finally, damping force connected with energy dissipation via acoustic waves has been considered. Resistant damping force can be calculated with the following formula [17]:

$$F = \rho\upsilon A\left\{1 - 2J_1\left(\frac{4\pi r}{\lambda}\right)\Big/\left(\frac{4\pi r}{\lambda}\right)\right\}\dot{x} \tag{7}$$

where υ is the speed of sound wave in the fluid (343m/s for air), λ is wave length of the sound and J_1 is first order Bessel function.

In Figure 8 bar diagram of all damping coefficients calculated for nominal configuration of FE model of MEMS resonator is presented.

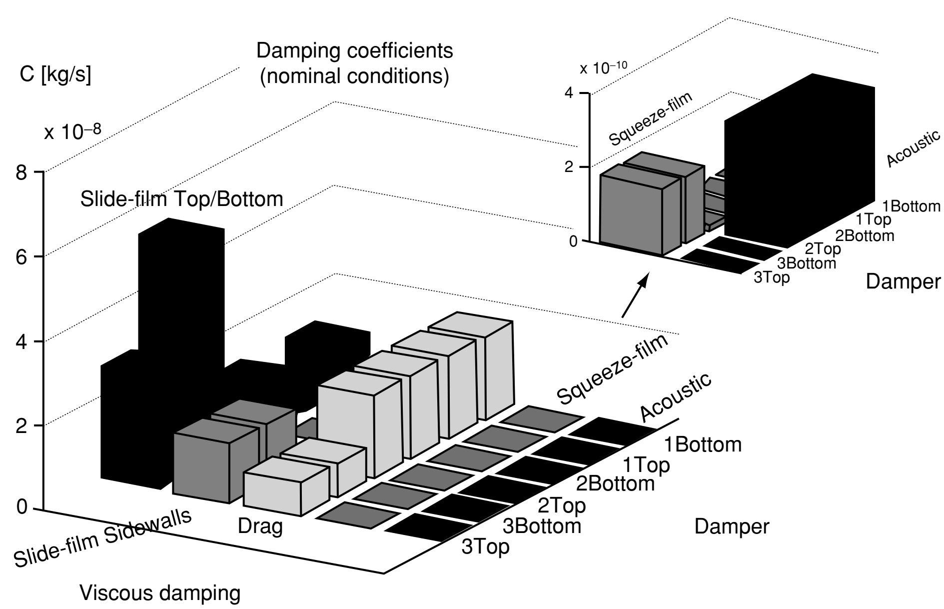

Figure 8 Damping coefficients calculated for nominal configuration of FE model.

Analyzing all calculated damping coefficients it can be observed that amongst all slide-film damping and drag forces characterize the greatest values, with values of 2 orders greater than those ones calculated for squeeze-film and acoustic wave dissipation. Therefore for elaborated model it seems to be enough to consider only first 2 kinds of sources of viscous damping. One can also notice that slide-film and squeeze-film damping mainly influence shuttle mass whereas drag force and acoustic dissipation mostly affect flexures.

3.2. ELECTROSTATIC FIELD IN FE MODEL

In the model each comb drive has been treated as the set of parallel capacitors. For each capacitor both normal and tangential forces exist and they both have been considered to express the presence of electrostatic field. In Figure 9 all considered capacitors are symbolically presented. They represent the sources of both tangential and normal electrostatic forces. Fringe effect observed for capacitors has been neglected in the study for the reason of simplicity of calculation [17].

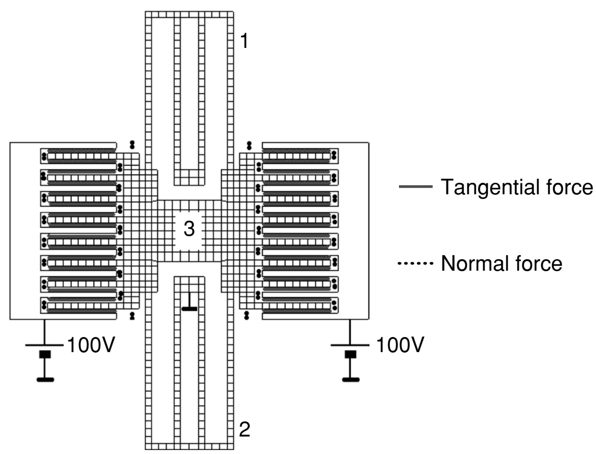

Figure 9 Areas of all considered capacitors and supplied constant voltages.

The following formulas can be used to calculate electrostatic forces:

a) tangential force:

$$F = \frac{b\varepsilon\varepsilon_0}{2d} V^2 \tag{8}$$

b) normal force:

$$F = \frac{A\varepsilon\varepsilon_0}{2} V^2 \frac{1}{x^2} \approx \left(\frac{\partial F}{\partial x}\right)_{x=x_0} \cdot x = kx \tag{9}$$

where b is thickness of comb drive finger, ε is relative permittivity of air (equals 1), ε_0 is permittivity of vacuum ($8.854 \cdot 10^{-12}$ F/m for air), V is voltage, d is gap between electrodes of horizontal or vertical capacitor, A is electrodes area of vertical capacitor, x and x_0 are respectively displacement and initial displacement of shuttle mass along axis x. Tangential forces calculated for all horizontal capacitors do not depend on the displacement x observed for assumed operational mode of vibration. Tangential forces are also constant because assumed applied voltages are constant. Moreover resultant tangential forces equals 0 as voltages have the same value and polarization. Therefore, described forces have not been taken in calculations. In case of normal forces, resultant stiffness coefficient can be calculated by the formula:

$$k \approx \left(\frac{\partial F}{\partial x}\right)_{x=x_0} = -A\varepsilon\varepsilon_0 V^2 \frac{1}{x_0^3} \tag{10}$$

which is derived from Eqn (9). Concluding, only tangential forces have been considered to determine approximated stiffness coefficients k of 2 springs connected to shuttle mass (location is marked as 3 in Figure 4). The sum of 2 nominal resultant coefficients k equals –0.256N/m. Negative value of k means that the introduction of electrostatic field lowers the resultant stiffness and resonance frequency of the structure vibrating with studied normal mode. Resultant stiffness of the model considered as one-degree-of–freedom system for considered normal mode and calculated for nominal configuration equals 52.0N/m.

For nominal conditions the comparison has been made between results obtained with elaborated model, in which air damping and electrostatic field have been considered in a simplified way, and results of simulation where coupled problem is solved (with ANSYS software). In case of calculations with MSC/Nastran the value of resonance frequency equals 136,962Hz whereas in case of coupled problem solution the value of studied characteristics is 136,499Hz. Relative difference equals 0.34%.

4. UNCERTAINTIES IN FE MODEL

For uncertainty analysis 65 input parameters have been considered. They represent variabilities of chosen geometric and material properties of FE model of MEMS resonator. In Table 1 information on uncertainty characteristics is presented. All input uncertainties have been modeled by fuzzy numbers with triangular and symmetrical shape of membership functions (presented in Figure 10). Assumed maximal ranges of variability correspond to intervals related to alpha-cut α_1 of input fuzzy numbers (where level of membership equals 0). All input parameters are treated as not correlated.

Table 1 Uncertain parameters

No. of parameter	Description	Nominal value	Maximal range of variability
1–16	Finger length (comb drives, fingers 1–16)	50μm	+/–0.5μm
17–32	Finger width (comb drives, fingers 1–16)	4μm	+/–0.1μm
33–48	Finger y-axis shift (comb drives, fingers 1–16)	0μm	+/–0.2μm
49–50	Flexure length (flexures 1, 2)	100μm	+/–1μm
51–60	Width of flexure beams (flexure beams 1–10)	4μm	+/–0.1μm
61	Microresonator thickness	3μm	+/–0.2μm
62	Chamfer angle of deposited layers (moving part only)	0 deg	0–1 deg
63	Young's modulus of polisilicon	165GPa	+/–3% (+/–5GPa)
64	Poisson's ratio of polisilicon	0.28	+/–10% (+/–0.028)
65	Mass density of polisilicon	2330kg/m^3	+/–3% (+/–70kg/m^3)

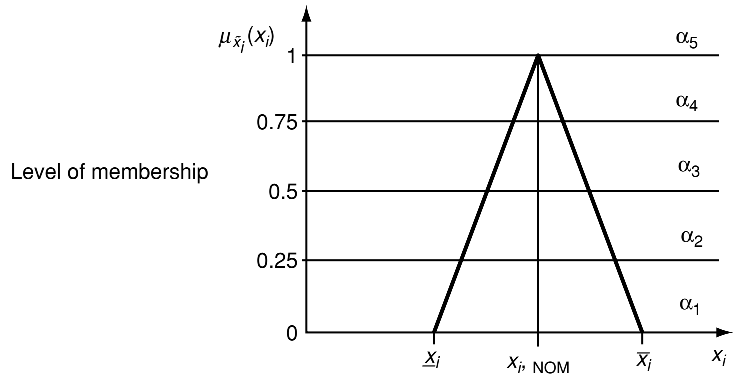

Figure 10 Input fuzzy numbers, membership function – 5 alpha-cuts considered.

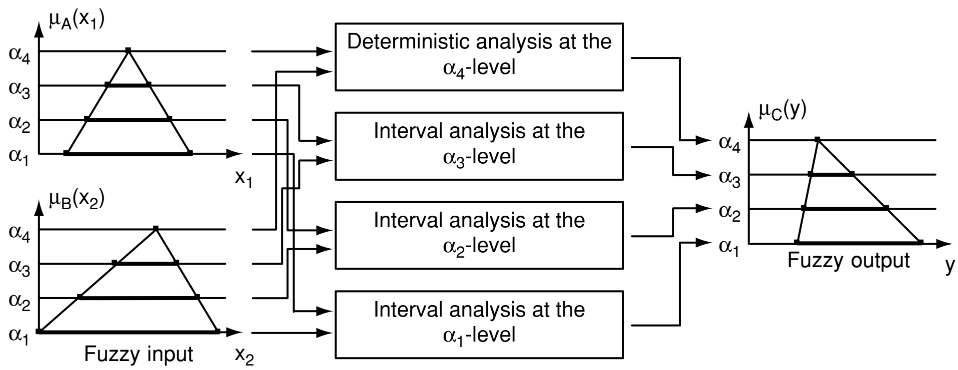

Figure 11 Idea of alpha-cut strategy (example considers only 4 alpha-cuts).

The alpha-cut strategy has been applied to calculate the output fuzzy numbers describing the variability of studied resonance frequency. The idea of this approach is presented in Figure 11 (example considers only 4 alpha-cuts). Described strategy means decomposition of original input fuzzy numbers into sets of intervals connected to particular alpha-cuts and then assembling the output fuzzy number with intervals determined during separate nondeterministic analyses performed for each chosen level of membership function.

In presented study 5 alpha-cuts have been considered. Alpha-cut α_5 is related to deterministic analysis that means nominal configuration of model input parameters. Output intervals (established by found extreme values) connected with alpha-cuts α_1, α_2, α_3 and α_4 have been achieved by the applications of:

a) MCS;
Uniform PDF has been assumed. Technique of Latin hypercube sampling [18] has been used to better the cover of input parameter domain with combination of values of uncertainties. 5000 samples per analysis have been considered.

b) GA [19,20];
Used applications of GA feature: 25 individuals, 120 generations, generation gap equals 0.8, probabilities of crossover and mutation equal 0.7 and 0.4 respectively. Two search tasks have been performed for each alpha-cut. First one for the search of minimal value and the second one to find the maximal value.

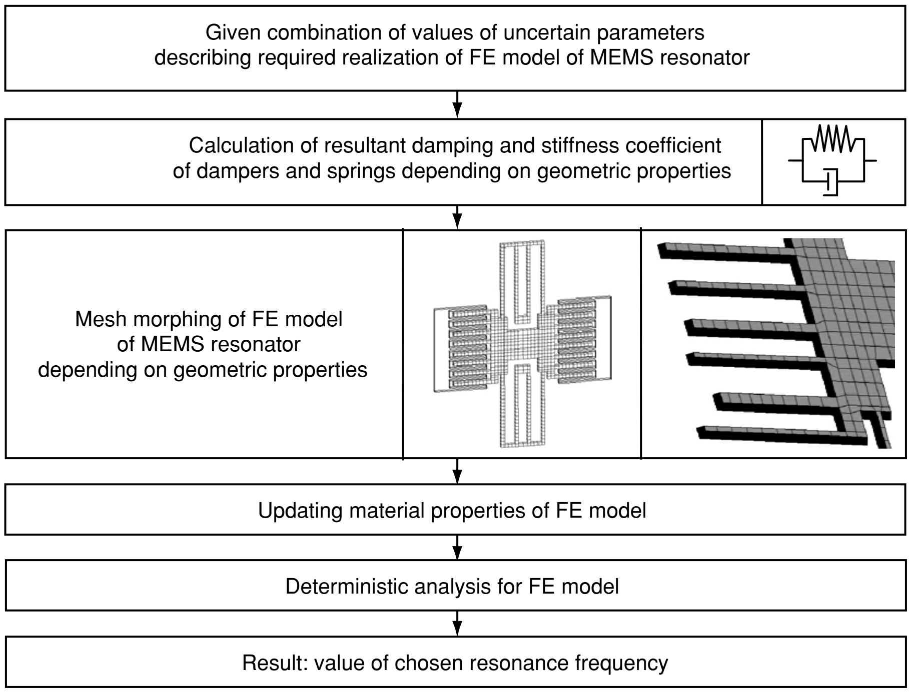

Figure 12 The scheme of simulation loop with deterministic calculations.

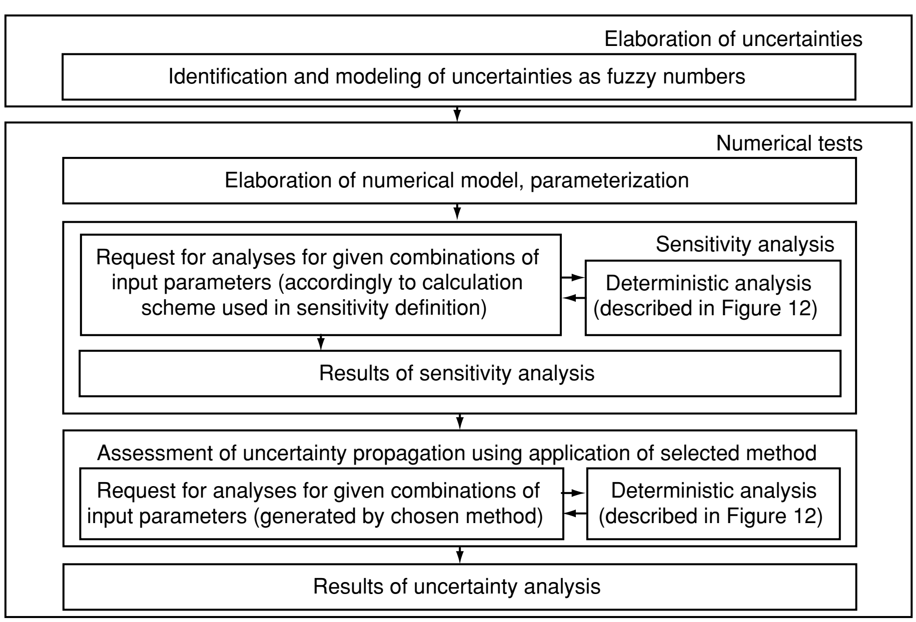

Figure 13 Uncertainty analysis performed for FE model of microresonator.

FE model of MEMS resonator has been parameterized considering all 65 uncertainties. The whole loop of deterministic calculations used for uncertainty analysis (for both sensitivity analysis and assessment of total uncertainty propagation by applications of MCS and GA) is presented in Figure 12. Performed deterministic calculations gives the results of one realization of FE model related to chosen combination of values of input uncertain parameters.

All calculations performed within sensitivity analysis and analysis of uncertainty propagation have utilized the loop described in Figure 12. Scheme used in mentioned above analyses is presented in Figure 13.

5. SENSITIVITY ANALYSIS

In presented work sensitivity analysis has been performed to find influences of all uncertain parameters on chosen resonance frequency. Finite difference method has been used as the approximation of first derivative [21]. The scheme of central plan has been applied for uncertainties apart from chamfer angle, where forward plan has been used. Results of sensitivity analysis are shown in Figure 14. Numbering of all microresonator parts presented in Figure 3 and Table 1 is again used in Figure 14. Presented sensitivities are normalized accordingly to nominal value of output parameter and calculated with the following expression (i.e. it stands for relative change of interesting parameter) [22]:

$$S_i = \frac{\Delta Y}{\Delta X_i} \cdot \frac{\Delta X_i}{f\left(X_{NOM}\right)} \tag{11}$$

where $\frac{\Delta Y}{\Delta X_i}$ is sensitivity (approximation of first derivative $\frac{\partial Y}{\partial X_i}$), ΔX_i means given interval of i-th input parameter (maximal range of variability according to Table 1), and $f\left(X_{NOM}\right)$ is value of output parameter for nominal combination of input uncertainties X_{NOM}.

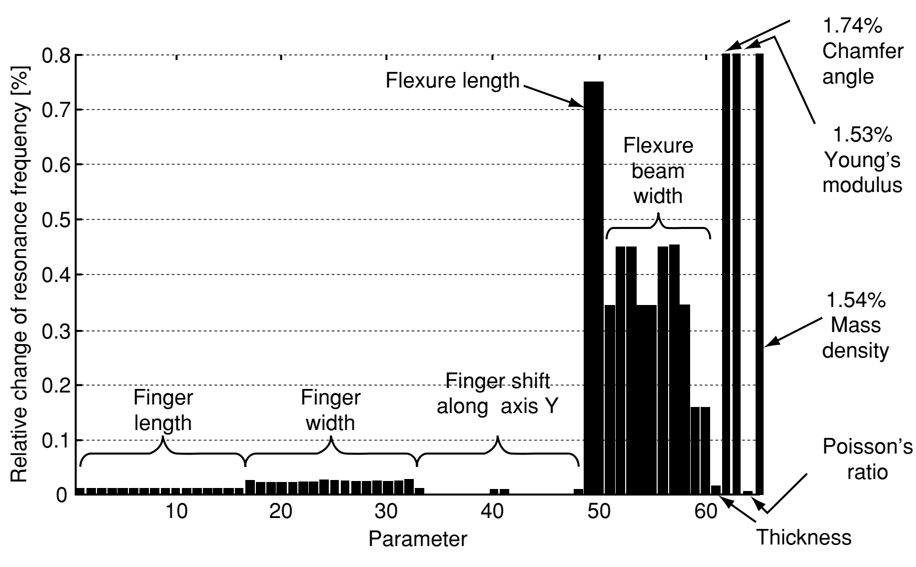

Figure 14 Calculated normalized sensitivities.

Considering given maximal ranges of variabilities of input parameters the most influential parameters are: chamfer angle (1.74%), mass density (1.54%) and Young's modulus (1.53%) of polisilicon as well as all 12 geometric properties of flexures. The influences of other parameters do not extend 0.05% of nominal value of studied frequency. Most influential 15 parameters should be then taken firstly under consideration during improving the performance of MEMS resonator. Studied mode of vibration characterizes negligible influence of thickness. This fact is understandable as increase of the value of this parameter causes both increase of the mass of moving part and increase of the stiffness of flexures that support it. Eventually, there is no significant difference in resonance frequency. All parameters that characterize geometry of comb drive fingers as well as Poisson's ratio of polisilicon can also be neglected during analysis.

6. UNCERTAINTY PROPAGATION

The propagation of all considered uncertainties has been assessed by the applications of MCS and GA. Detailed information on these applications can be found in section 4. Figure 15 shows plots of converges obtained in the application of GA for each considered alpha-cut and both while minimization and maximization processes. Presented values of resonance frequency are values of fitness function calculated for best fitted individuals.

Graphical representation of results obtained in uncertainty analysis is presented in Figure 16. Shown output fuzzy numbers representing variability of resonance frequency have been assembled using applications of MCS (market with dashed lines) and GA (marked with solid lines). Figure 16 also contains results of all realizations of FE model for applications of MCS (marked as signs +). Numerical form of yielded results is presented in Table 2.

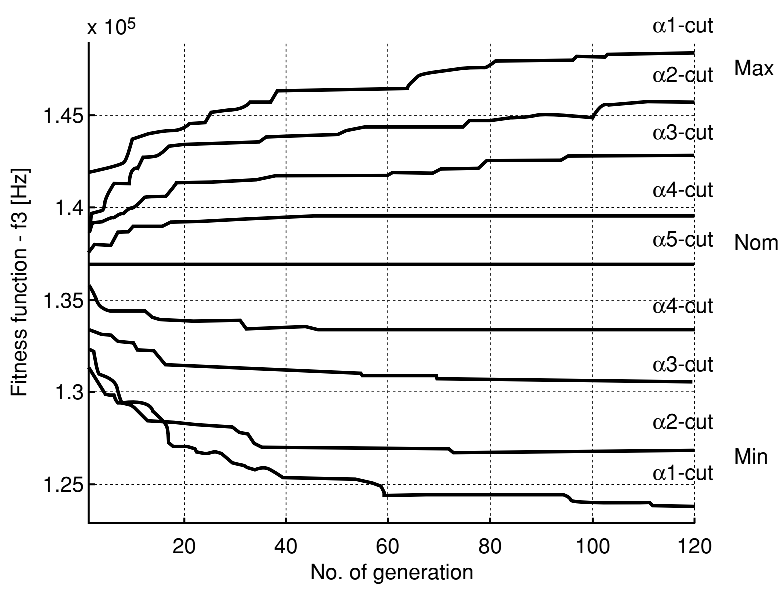

Figure 15 Plots of convergences obtained for applications of GA.

It can be noticed significant difference between results yielded by applications of MCS and GA. This observation can be explained by insufficient number of samples in MCS (5000; even with Latin hypercube sampling technique) used to cover uniformly and densely enough

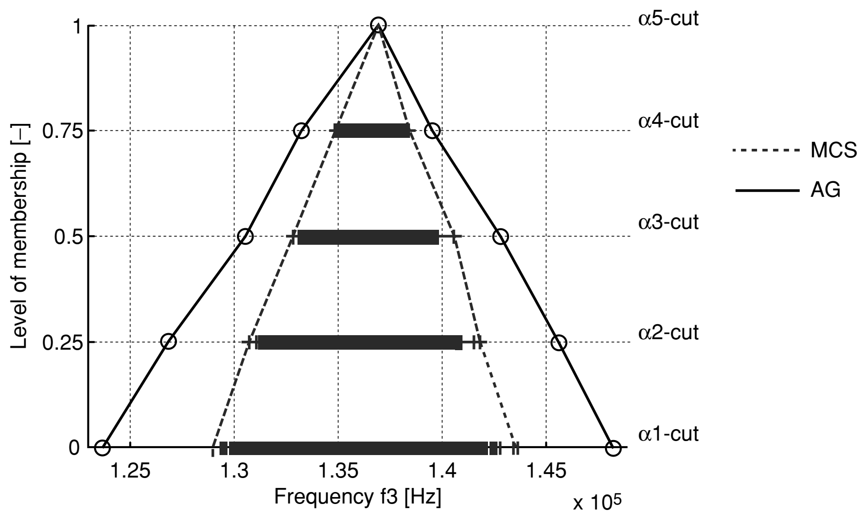

Figure 16 Uncertainty propagation for operational resonance frequency – output fuzzy numbers.

Table 2 Uncertainty propagation for operational resonance frequency – numerical representation

Parameter		α1-cut		α2-cut		α3-cut		α4-cut		α5-cut
		MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX	NOM
f_3	GA	123.7	148.3	126.9	145.7	130.6	142.8	133.3	139.6	137.0
[kHz]	MCS	129.0	143.7	130.8	141.8	132.8	140.6	134.9	138.4	

the domain of input parameters. Comparable number of iterations per each alpha-cut (4850) have been used to obtain extremes of output intervals while using GA. GA seemed to give more realistic results. However, there is still a justification of using MCS in such case. It can be applied to confirm that with high probability no global extremes has not been skipped while using application of GA.

7. CONCLUSION

In the work the application of uncertainty analysis is presented. As the object of the study chosen resonance frequency of operational normal mode of microresonator has been considered. Its variability has been assessed by means of fuzzy arithmetics which combined with alpha-cut strategy seems to be a powerful computational technique for analysis of uncertainty propagation. Obtained results can be used for inverse analysis as well, i.e. having output fuzzy number it is possible to predict what should be the maximal variabilities of input parameters to keep assumed range of variability of output characteristics, accordingly to chosen level of membership function. Probabilistic methods have been applied to find extremes of intervals of combined output fuzzy numbers, i.e.: MCS and GA. Noticed difference in yielded outcomes apparently appeared as a result of quite small number of samples in the application of MSC. 5000 iterations have not been enough to cover properly whole input domain but confirm that with high probability no global extremes has not been skipped by the application of GA.

For performed analysis a number of different kinds of uncertain parameters have been assumed representing both geometric imperfections and variabilities of material properties. Performed sensitivity analysis can be used while improving manufacturing process since influences of given uncertainties on studied characteristics have been found and most influential input parameters have been highlighted. As far as geometric parameters are of concern, flexure geometry and chamfer angle are most important for variability of studied frequency. It is so because the resultant stiffness for operational mode strongly depends on stiffnesses of flexures. Variability of resonance frequency is slightly influenced by changes of thickness of the device. Amongst material parameters only Poisson's ratio can be neglected because of its small contribution in changes of the value of resonance frequency.

In performed analyses FE model of MEMS resonator has been build and then parameterized accordingly to the list of all assumed uncertainties. The introduction of discrete mechanical elements allows for consideration of multiphysics in the FE model. Parameterized dampers and springs can effectively approximate the presence of air damping as well as electrostatic field. Amongst all studied damping phenomena slide-film damping and drag force should be of engineer's concern as they mostly influence calculated damping coefficients.

ACKNOWLEDGEMENTS

The work was supported by the Polish Grant no. N N503 141236, entitled "Metoda wielokryterialnej optymalizacji konstrukcji mikroukladow z uwzglednieniem niepewnosci technologicznych" ("The method of multiobjective optimization of the construction of micro devices considering technological uncertainties").

REFERENCES

[1] Van der Auweraer H., Requirements and opportunities for structural testing in view of hybrid and virtual modeling, *Proceedings of the International Conference on Noise and Vibration Engineering ISMA 2002*, Leuven, Belgium, September 16–18, 2002, 1687–1702.

[2] Lin R.M., Wang W.J., Structural dynamics of microsystems – current state of research and future directions, *Mechanical Systems and Signal Processing*, 2006, 20, 1015–1043.

[3] Liu M., Maute K., Frangopol D.M., Multi-objective design optimization of electrostatically actuated microbeam resonators with and without parameter uncertainty, *Reliability Engineering and System Safety*, 2007, 92, 1333–1343.

[4] Pitchumani M.R., Design of microresonators under uncertainty, *Journal of Microelectromechanical Systems*, 2005, 14(1), 63–69.

[5] Codreanu I., Martowicz A., Gallina A., Pieczonka L., Uhl T., Study of the effect of process induced uncertainties on the performance of a micro-comb resonator, *Proceedings of 4th Conference Mechatronic Systems and Materials 2008 - MSM 2008*, Bialystok, Poland, July 14–17, 2008.

[6] Martowicz A., Codreanu I., Uhl T., Uncertainty of design parameters and their influence on dynamic properties of MEMS microresonators. *Proceedings of 8th. World Congress on Computational Mechanics WCCM8 and 5th. European Congress on Computational Methods in Applied Sciences and Engineering ECCOMAS 2008*, Venice, Italy, June 30 - July 5, 2008.

[7] Codreanu I., Martowicz A., Uhl T., Influence of uncertain parameters on the performance of a micro-comb resonator. *Proceedings of 19th MicroMechanics Europe Workshop - MME 2008*, Aachen, Germany, September 28–30, 2008.

[8] Moens D., Vandepitte D., A survey of non-probabilistic uncertainty treatment in finite element analysis, *Computer Methods in Applied Mechanics and Engineering*, 2005, 194, 1527–1555.

[9] Oberkampf W.L., DeLand S.M., Rutherford B.M., Diegert K.V., Alvin K.F., Error and uncertainty in modeling and simulation, *Reliability Engineering and System Safety*, 2002, 75, 333–357.

[10] Moens D., Vandepitte D., Non probabilistic approaches for non deterministic FE analysis of imprecisely defined structures. *Proceedings of the International Conference on Noise and Vibration Engineering ISMA 2004*, Leuven, Belgium, September 20-22, 2004, 3095–3119.

[11] Schueller G.I., A state-of-the-art report on computational stochastic mechanics, *Probabilistic Engineering Mechanics*, 1997, 12(4), 197–321.

[12] Moore R.E., *Interval analysis*, Prentice-Hall, Englewood Cliffs, N. J., 1966.

[13] Dubois D., Prade H., *Fuzzy sets and systems. Theory and applications*, Academic Press, New York, 1980.

[14] Hanss M., *Applied fuzzy arithmetic. An introduction with engineering applications*, Springer-Verlag, Berlin, 2005.

[15] Hanss M., The transformation method for the simulation and analysis of systems with uncertain parameters, *Fuzzy Sets and Systems*, 2002, 130, 277–289.

[16] Mir S., Charlot B., Courtois B., Extending fault-based testing to micromechanical systems, *Journal of Electronic Testing: Theory and Applications*, 2000, 16, 279–288.

[17] Bao M., *Analysis and design principles of MEMS devices*, Elsevier Science, 2005.

[18] Helton J.C., Davis F.J., Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems, *Reliability Engineering and System Safety*, 2003, 81(1), 23–69.

[19] Goldberg D.E., *Genetic algorithms in search, optimization, and machine learning*, Addison-Wesley Publishing Company, Reading, Massachusetts, USA, 1989.

[20] Michalewicz Z., *Genetic algorithms + data structures = evolution programs*, Springer-Verlag, Berlin, Heidelberg, 1996.

[21] Kleiber M., Antunez H., Hien T.D., Kowalczyk P., *Parameter sensitivity in nonlinear mechanics*, John Wiley & Sons, Chichester, England, 1997.

[22] *Validation and updating of FE models for structural analysis. Theory, practice and applications*, Course materials, Dynamic Design Solutions NV, Leuven, Belgium, 2007.