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Numerical Simulation of fluidic aspects and heat transfer for geometry modification in an elliptical cylinder bundle

Dear Azizi[1], Mehrdad Malekzadeh Dirin[1*], Ramin Ghasemiasl[1]

[1]Department of Mechanical Engineering, West Tehran Branch, Islamic Azad University, Tehran, Iran.
*Corresponding author

Abstract

In this paper, flow field separation due to an adverse pressure gradient around rotating elliptical cylinders in a two-dimensional laminar flow inside a rectangular-shaped channel is investigated. To do so, the effects of the cylinder's rotation and aspect ratio are considered to examine the transient heat transfer, drag coefficient, velocity field, and vortex shedding. According to the results, it was reported that the maximum transient heat transfer occurred at Cylinder number 2, and the least heat transfer was related to Cylinder number 4. Thus, transient heat transfer increased by 33% by increasing the aspect ratio. The results show that the arrangement of elliptical cylinders inside the channel has caused the minimum pressure drop in the flow direction. The vortex generation frequency is also reported to be the least for the first cylinder located along the channel, and by moving from the channel inlet, the magnitude of these vortices becomes larger. The maximum drag force is imposed on Cylinder number 5 (the last cylinder), while, contrary to expectations, the least drag force is applied to Cylinder number 2. Statistically, by increasing the rotation angle from Cylinder number 2 to Cylinder number 5, the skin friction coefficient increases by 107%, denoting the increase in drag force. It is also reported that the vortex distribution is symmetrical around both sides of the cylinders.

Keywords: Numerical analysis, Convective heat transfer, Smooth flow of fluid, Elliptical cylinders, Circular cylinders.

1. Introduction

Elliptical cylinder bundles are widely used in various industries for applications such as heat exchangers and reactors [1]. It is essential to ensure effective heat transfer between the fluid and the surrounding environment in these applications. The performance of elliptical cylinder bundles is influenced by fluid dynamics, which can be optimized by modifying the geometry of the cylinders within the bundle [2]. One way to enhance heat transfer is by changing the shape, size, and arrangement of the cylinders to improve flow patterns and fluid dynamics [3]. For example, adjusting the spacing between the cylinders can reduce flow resistance, promote better fluid mixing, and increase heat transfer efficiency [4, 5]. Additionally, modifying cylinder orientation or introducing baffles can control flow direction and enhance heat transfer rates. Another method to improve heat transfer in elliptical cylinder bundles is by modifying the geometry of the cylinders themselves [6]. Adding fins or protrusions to the cylinder surface can increase the available surface area for heat transfer, leading to more efficient heat exchange with the fluid. Altering the aspect ratio or eccentricity of the cylinders can also impact heat transfer characteristics and flow behavior within the bundle [7]. In this regard, Vijay et al. [8] carried out to study the effect of transverse vibration originated from an induced flow over an elliptical cylinder with various aspect ratios. The cylinder was supposed to be confined to a vibration in the flow direction and Reynolds number was supposed to be constant and equal to 100. Also, aspect ratio varied from 0.1 to 1. According to the results, the response of elliptical cylinder with least aspect ratio of 0.1 is reported to be twice over than that of circular cylinder. Karlson et al. [9] conducted a study on the vortex length generated in a flow over an elliptical cylinder. Aspect ratio ranging from 0.4 to 1.4 and Reynolds number ranging from 10 to 100 were considered. They found that, in a steady state condition, variation of vortex length with regard to Reynolds number is a linear trend while, in unsteady condition, the mean length of vortex tends to exponentially decrease by increasing the Reynolds number. Nag et al. [10] carried out to numerically investigate the natural convection flow with two Newtonian and non-Newtonian fluids flowing over an isothermal elliptical cylinder. Modified viscous law model was also employed to determine the characteristic of the non-Newtonian fluid. Also, local friction coefficients and heat transfer rate from cylinder surface (local Nusselt number) were taken into account as a function of elliptical cylinder angle with different aspect ratios. It was reported that the local friction coefficient of the considered non-Newtonian fluid is superior than that of Newtonian fluid. Overall heat transfer rate of the elliptical cylinder with weak positioning is greater than the firm positioning.

Similarly, Seo et al. [11] investigated the effect of elliptical cylinder dimensions located in a rectangular channel on the heat transfer rate. Immersed boundary method was used to define the cylinder wall boundary through the Finite Element approach. The range of Reynolds number was considered between 10000 to 1000000. While flow over circular cylinders has been extensively studied, there is a lack of research on flow over elliptical cylinders. Therefore, studying flow over elliptical cylinders can fill this research gap and provide a more comprehensive understanding of fluid dynamics. Elliptical cylinders are commonly used in engineering applications, such as heat exchangers and nuclear reactors, and understanding their flow characteristics is important for optimizing their design and performance. Flow over elliptical cylinders can be considered prototypical of flow over a range of bluff bodies, making it a useful model for studying the effect of both geometry and flow parameters on fluid dynamics. Understanding the phenomenon of flow separation and bluff body wakes around elliptical cylinders is fundamental to flow physics and has practical applications in fields such as aerodynamics and fluid mechanics. Therefore, the aim of this study is to investigate the thermo-fluid interactions of a flow over an elliptical rotating cylinder located in a rectangular channel. Also, the effect of the presence of such rotating cylinders on the parameters such as heat transfer rate, flow separation over the cylinder, vortex shedding, pressure drop and drag force is considered. Shi et al. [12] conducted a study to explore the three-dimensional flow patterns of a wavy elliptic cylinder and its wake for different ratios of wavelength to hydraulic diameter. They applied time-averaged streamlines and critical point theory to observe the three-dimensional flow structures. They reported that spiral flows, counter-rotating vortices, and wavy vortex structures were observed at lower ratios of wavelength to hydraulic diameter. While, bifurcation in stable recirculation and vortex shedding occurred at higher aspect ratios. In a similar study, they [13] repeated their study but at a low Reynolds number. They found that increasing the wavelength leads to a decrease in the strength of the wake vortices shed by the cylinder, resulting in a more stable and less turbulent wake. Kukreti et al. [14] considered different aspect ratios to see the fluid flow behavior passing through an elliptical cylinder located near a moving wall. In their study, the key flow characteristics were wake pattern, separation and stagnation points, and hydrodynamic forces. Their results showed that as aspect ratio increases, the flow structures and vortex shedding patterns change, leading to significant variations in the lift and drag forces experienced by the cylinder. They also reported that the positions of where separation and stagnation occur are determined by the interaction of aspect ratio and the suppression of wall effects, showing a decreased sensitivity to Reynolds number when aspect ratio is low or when wall suppression is high. Duong et al. [15] focused on the effects of the cylinders' spacing, orientation, and aspect ratio on flow characteristics such as wake structure, drag coefficient, and lift coefficient. The found that the wake structure and flow patterns are influenced by these parameters, highlighting the importance of geometric configurations in controlling the flow behavior around tandem elliptical cylinders. Li et al. [16] described a numerical study using large eddy simulation to investigate the flow around an elliptic cylinder with various aspect ratios in the subcritical regime. With aid of 2D and 3D observations of wake patterns, they illustrated the variations in vortex shedding modes and the features of vortex structures at different aspect ratios. According to their outcomes, it was shown that the aspect ratio significantly influences the flow patterns and drag coefficients of the cylinder, with smaller aspect ratios leading to higher drag coefficients and more complex flow structures.

Wu et al. [17] carried out to investigate the flow-induced vibrations of an elliptic cylinder for which a splitter-plate attachment was used at low Reynolds numbers. They reported that the splitter-plate attachment causes self-limited oscillations of the cylinder, with the amplitude of the oscillations remaining within a certain range. They also concluded that the splitter-plate attachment can be an effective way to control flow-induced vibrations in certain scenarios. In a study conducted by Gupta et al. [18], rotating elliptical cylinders circumscribed in a narrow enclosure was numerically investigated. In their research, the enclosure’s walls were considered as confinement, prohibiting the flow over rotating elliptical cylinders in the transverse direction. Through a parametric study, parameters such as confinement ratio, nondimensional rotation rate and the Reynolds number were considered to identify the variations in flow's mechanical properties like drag-coefficient and moment coefficient in addition to investigating the shedding of vortices. They reported that the presence of confinement would lead to delay in shedding of vortices however higher confinement ratio could not fully suppress the vortex formation due to sharp flow separation at the edges of rotating elliptical cylinders. Similarly, Kumar et al. [19] carried out to examine the forced convection heat transfer characteristics of an unconfined power-law fluid flow over rotating elliptical cylinders. Their research focus was placed on parameters such as rotational speed, power-law index, cylinder aspect ratio and Prandtl number in order to examine the heat transfer characteristics in laminar flow field and compare their results with circular cylinders. According to their results, elliptical configuration showed superior heat transfer characteristics (higher Nusselt Number), when compared with its circular peer. Kumar et al. carried out a similar study with non-Newtonian (power-law) fluid flow over a set of rotating elliptical cylinders. They reported that increase in rotational speed and power law index has a positive impact on wake formation at the rear of cylinders. Taloub et al. [20] carried out to investigate the impact of buoyancy force on natural convection heat transfer over a set of elliptical cylinders. They found that there is a certain value for buoyancy force for which mean Sherwood and Nusselt number values are minimal. In fact, there is a threshold for buoyancy force that says if buoyancy force value is below the critical value the natural convection heat transfer decreases by increasing the buoyancy force, and if the buoyancy force is higher than critical value the heat transfer increases by increasing the buoyancy force. Farokhi et al. [21] investigated the geometrical parameters of a laminar flow over two tandem elliptical cylinders in a way that aspect ratio of upstream varied linearly while the downstream aspect ratio was considered constant. Their results indicated that there is a significant discrepancy of 178% in the estimation of the hydraulic diameter of the elliptic cylinder, which greatly impacts the lift and drag coefficients. They also concluded that the phase lag between the sinusoidal lift coefficients of the cylinders varies, reaching its minimum at an aspect ratio of 1.5 and slightly increasing at an aspect ratio of 2.0. According to their outcomes, it was observed that the minimum Stanton number of 0.065 occurs at an aspect ratio of 0.25 and gradually increases to a maximum value of 0.211 at an aspect ratio of 1.75.

The research gap regarding the flow over rotating elliptical cylinders primarily lies in the lack of comprehensive understanding and analysis of the fluid dynamics involved in such flow phenomena. According to the literature, while extensive research has been conducted on flow over elliptical cylinders, the investigation of flow characteristics particularly convective heat transfer over rotating elliptical cylinders remains relatively limited. Another significant research gap in this field is the limited understanding of the flow patterns and boundary layer behavior over rotating elliptical cylinders. The complex geometry of elliptical cylinders, with different curvature radii at each axis, introduces intricate flow patterns and aerodynamic characteristics that differ from those observed in circular or rectangular cylinders. Therefore, the investigation of the effects of rotation on the flow over elliptical cylinders becomes crucial for understanding the underlying physics and developing accurate predictive models. Finally, literature has not fully covered the impact of varying parameters like aspect ratio on the flow field over rotating elliptical cylinders. Understanding how such parameters govern the formation of vortical structures, shedding frequencies, and drag coefficients is essential for practical applications such as designing energy-efficient devices, optimizing aerodynamic performance, and preventing unfavorable flow conditions like vortex-induced vibrations. Thus, this research goes beyond previous studies in the field of flow over a bundle of elliptical cylinders by focusing specifically on the thermo-fluid interactions of fluid flow over rotating elliptical cylinders in a rectangular channel. One key novelty of this research is the focus on geometry modification in the elliptical cylinder bundle. In this study, we manipulate different aspect ratios of elliptical cylinders to identify the optimum states of fluidic aspects and heat transfer in the elliptical cylinder bundle. Additionally, we aim to investigate the effects of rotating cylinders on various parameters such as heat transfer rate, flow separation, vortex shedding, pressure drop, and drag force. Finally, the outcomes of this research offer a unique contribution to the existing literature on flow over a bundle of elliptical cylinders.

2. Model description and governing equations

2.1 Model description

This study focuses on using numerical simulation techniques to investigate the flow characteristics of laminar convection over a bundle of elliptical cylinders. The aspect ratio of the cylinders is varied to examine its effect on the flow behavior. The study aims to enhance the understanding of the fluid flow patterns, heat transfer, and resistance coefficients associated with convection over bundles of elliptical cylinders. The numerical simulation method employed in this research involves solving the Navier-Stokes equations combined with the energy equation, which accounts for the heat transfer mechanism [22, 23]. The significance of this study is the ellipses that have significant impact on heat transfer, particularly when compared to circular cylinders. By modifying the aspect ratio, the shape of the elliptical cylinder's changes, which, in turn, affects the flow conditions. Therefore, investigating how varying the aspect ratio influences the flow behavior will provide valuable insights for a wide range of engineering applications, such as heat exchangers, cooling systems, and fluid transport systems [22]. Simulations are carried out under various aspect ratios, and the resulting flow characteristics, heat transfer patterns, and friction coefficients are analyzed, compared, and discussed in detail. More to the point, in this paper, a rectangular two-dimensional channel in which 5 elliptical cylinders are positioned is considered. The channel is positioned horizontally, and it has 23 units in length which is started from -12 in coordinate system and ended at the position of +11 in positive axis. Elliptical cylinders are located at the first half of the left side of the channel and the line of y=0 is aligned with the center of these cylinders. The width of the channel is 4 units long that the concentric cylinders are in line with the middle point of the channel's width at coordinate of 2. The first cylinder is located in 1.5 units far from the channel inlet and the distance between the concentric cylinders is the same and equal to 2. The rotation angle of each cylinder against the previous one is 22.5 degrees, and since it is started from zero angle for the first cylinder positioned, the rotation angle for cylinder number 5 becomes 90 degrees. Also, it should be noted that the elliptical cylinders are the same in size, each of which has a large diameter of 1.4 units and small diameter of 0.7. Physical characteristics as well as dimensional coordinates of the elliptical cylinders arranged in a rectangular and horizontal channel is depicted in Figure 1.

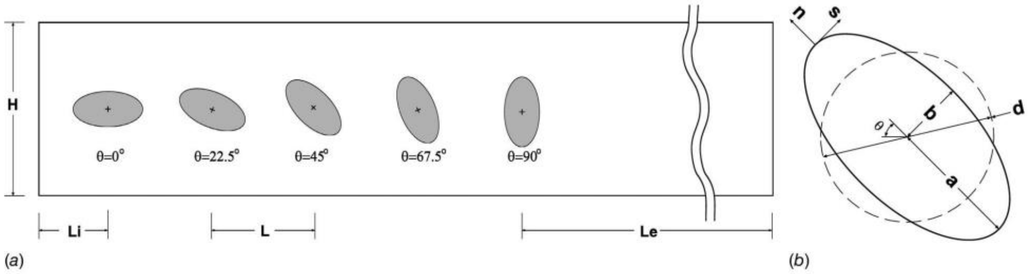

Figure 1. A schematic of a horizontal channel along with the elliptical channels.

Dimensions used in this study are based on the Ref [24], and the aspect ratio is initially considered to be 0.5. The geometry and the arrangement of the elliptical cylinders is set to impose minimum pressure drop and have maximum heat transfer rate at the same time. To do so, the gradual rotation of the cylinders in the flow direction is applied. In order to determine the effect of aspect ratio of the elliptical cylinders, the flow over the elliptical cylinders with different aspect ratio ranging from 0.45 to 0.55 which was authenticated by Ref [24] and over this value is considered. Figure 2 indicates a schematic of the given geometry with different aspect ratios. Also, physical characteristics and dimensional coordinates of this geometry is summarized in Table 1. It is also noticeable to mention that the point in changing the aspect ratio of the current elliptical cylinder before defining its characteristic diameter is to study the effects of different aspect ratios on the cylinder's properties. By changing the aspect ratio, which is the ratio of the major axis length to the minor axis length of the ellipse, we can investigate how the cylinder's shape and behavior change. In fact, this non-dimensional consideration allows for a more comprehensive understanding of the flow behavior and facilitates comparisons with other geometries. This can be useful in various fields such as engineering, fluid dynamics, and materials science, where understanding the impact of different geometries is important. On the other hand, by defining the characteristic diameter of the elliptical cylinder with different orientations, we can quantify its size and compare it to other

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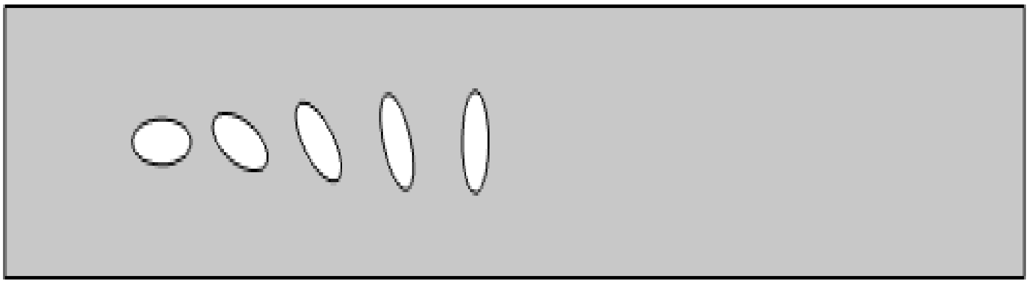
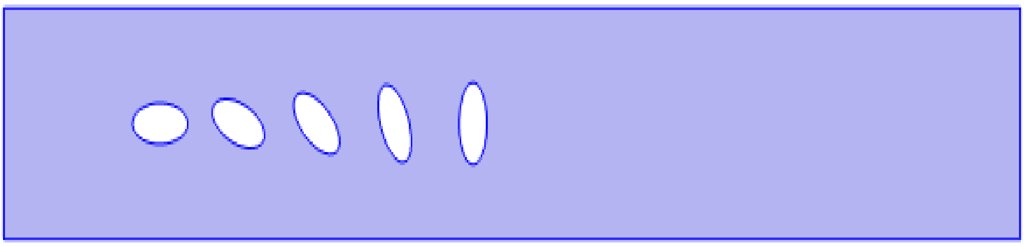
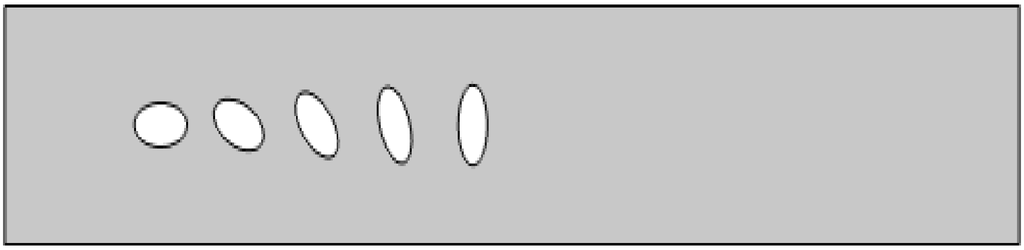
cylindrical shapes. This allows for a standardized way to assess and compare different elliptical cylinders in terms of their dimensions and properties.



Aspect ratio of 0.45



Aspect ratio of 0.5



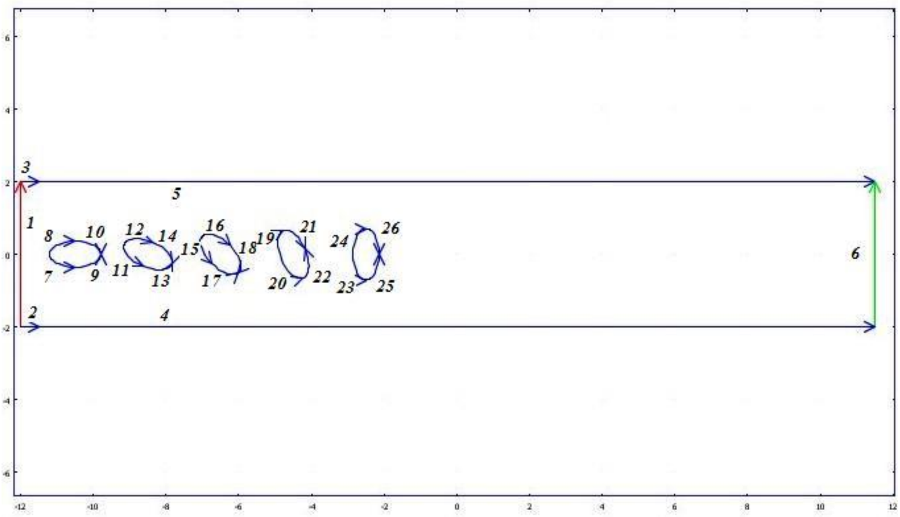
Aspect ratio of 0.55

Figure 2. A schematic of rectangular channel along with the elliptical cylinders with different aspect ratios.

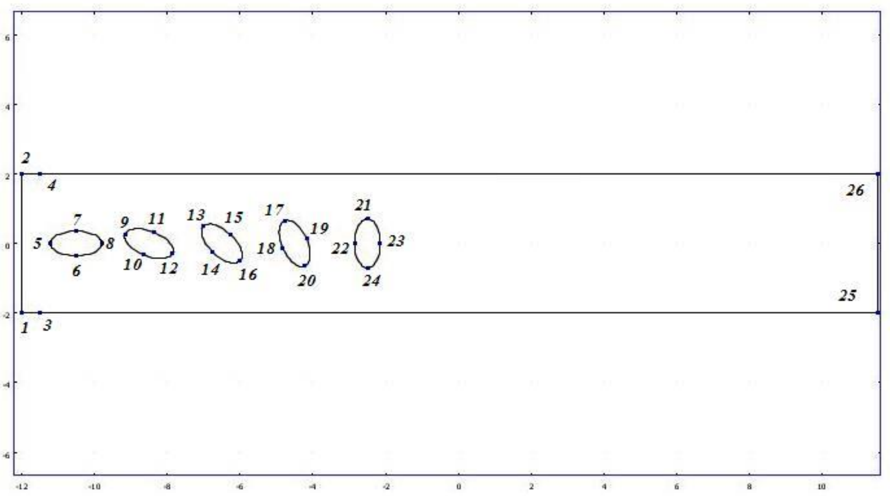
Table 1. Physical characteristics and dimensions of the elliptical cylinders.

a	b	C	d	E	f	g	
Small diameter of elliptical cylinder	Large diameter of elliptical cylinder	Width of the channel	Length of the channel	Distance from first cylinder with the channel's inlet	Distance from the cylinder with the channel's horizontal line	Distance between two elliptical cylinders	**Title**
0/7	1/4	4	23	1/5	2	2	**Size**

The geometry of the elliptical cylinders used in this study consists of 26 boundary lines which is depicted in Figure 3 (A). These boundary lines are considered as a part of the channel or are the curves which has surrounded the elliptical cylinders. As can be seen from the fig. 3, the channel's inlet and outlet cross sections are defined by the line number 1 and 6, respectively. Also, each elliptical cylinder is composed of 4 numbered curves and attributed to the four points, each of which is also attributed to a certain number. The inlet and outlet regions of the channel are outlined by using points number 1, 2 and 7, 8, respectively. To have a better prospective of what is stated, Figure 3 (B) is sketched.



A: Boundary lines



B: Boundary points

Figure 3. A schematic of the boundary lines and boundary points of the rectangular channel along with the elliptical cylinder.

2.2. Governing equations

Generally, the principal equations of flow over a series of cylinders are governed by partial differential equations which are derived from the mass balance and momentum (Navier-Stocks) equations. For a steady state, viscous and incompressible flow, momentum and continuity equations are solved to determine the velocity field and pressure distribution along the channel according to the given boundary conditions. Since the rectangular channel is considered tow dimensional, the governing equations are presented in the form of 2D [25].

For Mass conservation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{Eq. 1}$$

For x-Momentum conservation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{Eq. 2}$$

For y-Momentum conservation:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}) \quad \text{Eq.3}$$

For Energy conservation:

$$\frac{\delta T}{\delta t}+(u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y})=k/\rho C_p(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}) \quad \text{Eq. 4}$$

Where in above equations, u and v are the velocity profile in the x and y directions, respectively. Also, ρ is the indication of density, P is the fluid pressure, μ is the dynamic viscosity and T is Temperature. In this paper, it is assumed that the flow is incompressible. Equations 1 to 3 were used by COMSOL software for a computational domain, shown in Figure 1. Also, the parameter "aspect ratio" is defined by the following equation.

$$AB=\frac{b}{a} \quad \text{Eq. 5}$$

Where a is the diameter of the larger side of the elliptical cylinder and b is the diameter of the smaller side. Taking this expression into account, parameters such as Reynolds number, Drag and Lift forces are defined by the following equations.

$$Re=\frac{\rho uD}{\mu}=\frac{2ua}{\upsilon} \quad \text{Eq. 6}$$

$$C_D=\frac{F_D}{\frac{1}{2}\rho u^2 a} \quad \text{Eq. 7}$$

$$C_L=\frac{F_L}{\frac{1}{2}\rho u^2 a} \quad \text{Eq. 8}$$

To perform the numerical calculations, some parameters should be obtained beforehand. These parameters are such as subspace fluid characteristics, stabilization of subspace equations and initial values. Properties of subspace fluid is determined by the density, dynamic viscosity, bulk viscosity and bulk forces in the X and Y direction. Values of these coefficients are summarized in Table 2. Equations dealing with thermo-fluid subspace system are obtained below.

Table 2. Fluid characteristics related to thermo-fluid subspace module.

Title	Indices	Unit	Value
Density	ρ	Kg/m^3	**1.2**
Dynamic viscosity	η	Pa.s	**1.5** e^{-3}
Bulk viscosity	K_{dv}	Pa.s	**0**
Bulk forces in the X direction	F_x	N/m^3	**0**
Bulk forces in the Y direction	F_y	N/m^3	**0**

$$\nabla.\Gamma=F \quad \text{Eq. 9}$$

$$c=-\frac{\partial\Gamma}{\partial\nabla u} \quad \text{Eq. 10}$$

$$a = -\frac{\partial F}{\partial u} \quad \text{Eq. 11}$$

$$\gamma = \Gamma \quad \text{Eq. 12}$$

$$\beta = -\frac{\partial F}{\partial \nabla u} \quad \text{Eq. 13}$$

$$\alpha = -\frac{\partial \Gamma}{\partial u} \quad \text{Eq. 14}$$

$$f = F \quad \text{Eq. 15}$$

Where in above equations, c is the diffusion coefficient, a is the absorption coefficient, f is the source term, β is the convection coefficient, Γi s circulation.

3. Numerical simulation

In this study, COMSOL software was used with aid of Finite Element method to perform the numerical calculations of the Navier-Stocks and heat transfer equations. For turbulent condition, the terms of turbulency are added to these equations. Here, $k-\varepsilon$ model was used to consider the turbulency effects. Numerical calculations were based on unsteady and transient conditions. Also, time step was wisely selected in a way that the residuals remain at least and governing equations are converged in every single time steps. To do so, variable time step method was then applied in a way that the COMSOL software automatically adjusts the time step. When turbulent flow is the case, time step is usually around 0.001. For all studied cases, simulations continued until initial fluctuations of the flow disappear and flow reaches quasi steady state or periodic condition. Also, fluid characteristics and inlet velocity were considered as initial boundary conditions as well as an indication of equivalent inlet Reynolds number. Prandtl number was also set to be 0.71 which is the same as the air Prandtl number.

3.1. Meshing

When parameters such as coordinates, subspaces and boundary lines are determined, it is the time to determine the elements of which all governing equations are solved and iterated. Meshing and the size of the elements can change according to the required accuracy of the results of a specific problem. Although performing fine and ultra-fine meshing can enhance the results, there should be an attention to the computational equipment and costs associated. Hence, by taking two indicators like cost and accuracy, mesh independency study was carried out. In this study, triangular elements were considered. Figure 4 indicates a view of the meshing elements.

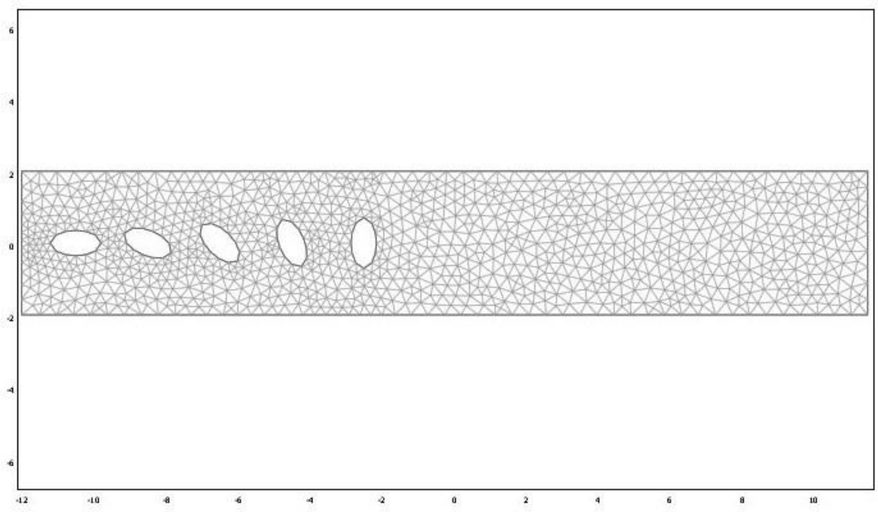


Figure 4. A schematic of the meshing elements in a rectangular channel along with the elliptical cylinders.

In this paper, meshing which is unstructured and triangular prism was performed by using the tools in COMSOL environment. Generally, element size along the channel was considered to be fine, while the meshes around the cylinders were boundary layer type meshing and ultra-fine. This consideration caused a better prediction of flow field and heat transfer around the cylinders. Also, the total number of elements in the computational domain was near 160000 triangular elements that is supposed to solve governing equations in each time step. A zoomed view of sample generated mesh of one cylinder in the computational domain is shown in Figure 5.

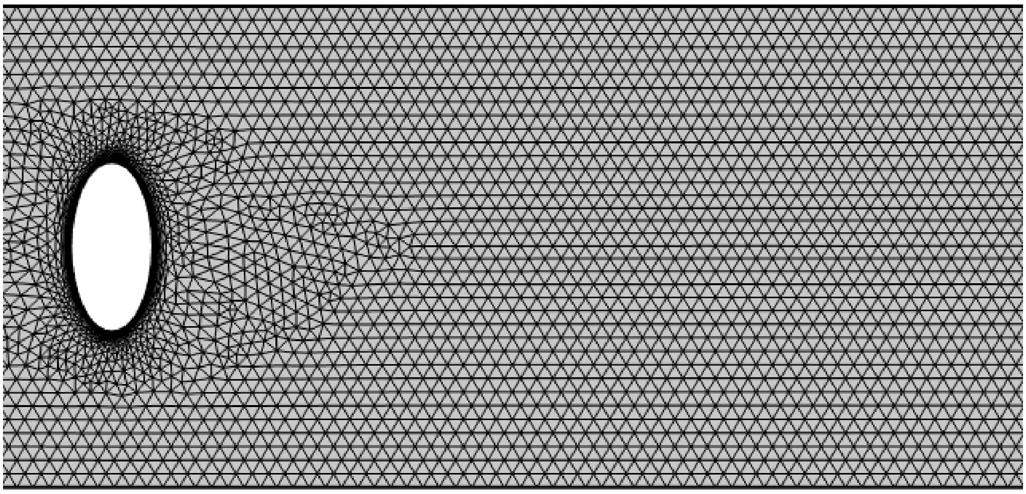


Figure 5. A zoomed view of generated fine mesh.

3.2. Mesh independency study

To conduct the mesh independency study, first different types of element size including ultra-fine, fine, normal, coarse and ultra-coarse with triangular shape were considered to study the temperature distribution. Then, the results of all configurations are compared to determine the optimum state of element size configuration. The results of mesh independency study of all element size configurations for mean Nusselt number variation against time on the last cylinder is shown in Figure 6. As can be seen from Figure 6, the behavior of Nusselt number varies by increasing the element size while Nusselt number has shown no significant changes for fine element size compared with ultra-fine element size. Therefore, it can be concluded that the fine element size is a good candidate for current study since it implies good accuracy and lower computational time as the same time.

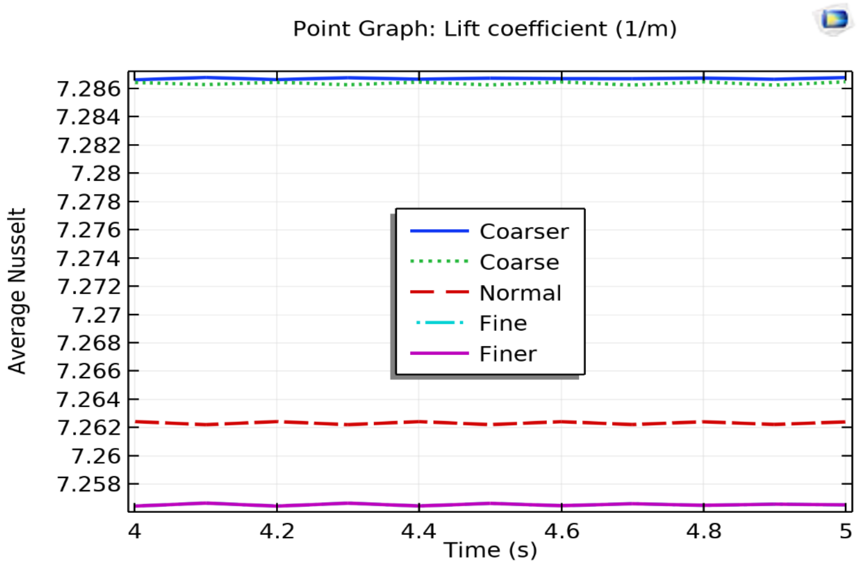


Figure 6. Mean Nusselt number variation against time for different element size configurations.

4. validation

To make sure that the numerical calculations are accurate, the results obtained from current study are compared with their peers obtained by other scholars. Since studies dealing with the flow over the elliptical cylinders are not ubiquitous, current achievements are compared with results obtained from Ref [24]. It should be mentioned that both laminar and turbulent flows are validated.

4.1. Comparison of the numerical results with the results obtained by Ref [24]

In this section, the results of current study on a rectangular channel along with the elliptical cylinders are compared with the results obtained by Ref [24]. For comparison, contour of velocity field along a complete vortex shedding cycle was used. The figure of velocity stream lines for Reynolds number of 500 obtained from the current study is compared with their peers obtained by Ref [24], and is shown in Figure 7. As can be seen from the, current study has shown to have good agreement with the data provided by Ref [24].

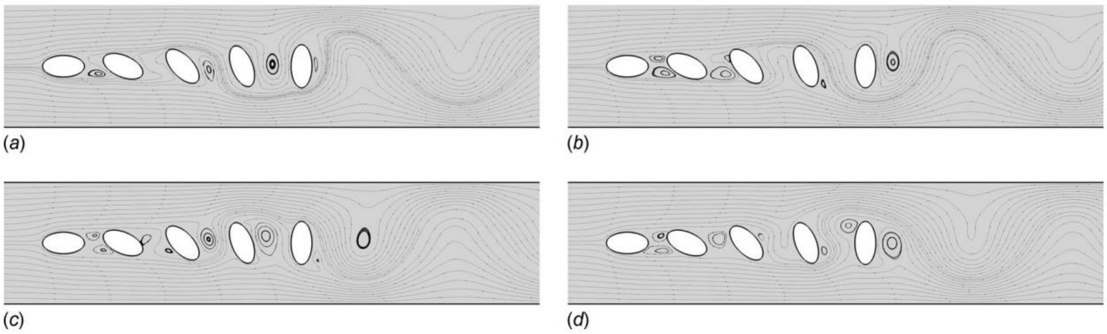


Figure 7. Validation of velocity stream line of the current study against the results obtained from Ref [24].

4.2. Validation of laminar flow

In this section, the results of current numerical study on the laminar flow are compared with their peers obtained by Ref [24]. According to assumptions made by Ref [24], here, Reynolds number was considered to be 100 and 200, and aspect ratio was equal to 0.5. To report the validation results, Nusselt number as a heat transfer criterion and Strouhal number as a flow fluctuation criterion inside the channel was selected, and results obtained from other studies have been summarized in Table 3 for comparison.

Table 3. Results of Nusselt and Strouhal number obtained from validation of the current study compared with their peers.

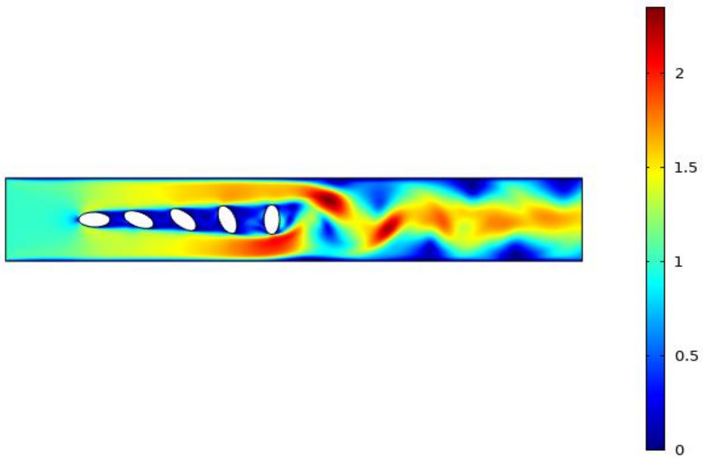
	Strouhal number		**Nusselt Number**	
Reynolds number	Re=100	Re=200	Re=100	**Re=200**
Current study(Num.)	0.164	0.196	5.182	**7.220**
Zhuauskas [26]	-------	-------	5.1	**7.212**
Knudsen and Katz [27]	-------	-------	6.294	**7.197**
Churchill and Benstein [28]	-------	-------	6.292	**7.228**
Alawadhi [25]	-------	-------	5.184	**7.221**
Roshko [29]	0.16-0.17	0.17-0.19	-------	**-------**
Norberg [30]	0.168	0.18-0.197	-------	-------
Williamson [31]	0.164	0.196	-------	**-------**
Meneghini et all [32]	0.162	0.196	-------	-------
Braza et all [33]	0.16	0.2	-------	**-------**
Ding et all [34]	0.166	0.196	-------	**-------**
Alawadhi [25]	0.163	0.196	-------	**-------**

5. Investigation on laminar flow within the channel with aspect ratio of 0.5

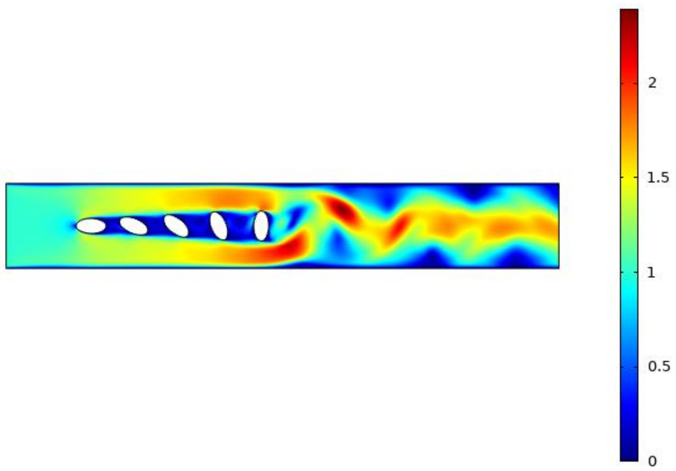
In this section, results obtained by numerical calculations of a laminar flow over the elliptical cylinders inside a rectangular channel with aspect ratio of 0.5 are investigated. Also, the effect of elliptical cylinder arrangement along the channel on the vortex shedding, vortex formation, drag and lift coefficients along with the variations of these parameters over time (dimensionless time) is investigated. The following results are obtained by coupling the fluid flow equations with those of heat transfer along the channel where the flow was laminar. Figure 8 indicates variations of dimensionless velocity contour against dimensionless time. It should be also mentioned that the vortex shedding time is supposed to be 0.3 seconds so that the parameter of dimensionless time is based on this value.

Since the Reynolds number is relatively low, term of viscous forces overcomes the inertia forces, thus it causes a symmetrical flow over both sides of the cylinder and stream lines would not be separated from the cylinder surfaces. While increasing the inertia forces and consequently increasing the Reynolds number would intensify the term of turbulency and then viscous forces supersedes which all makes the flow separated. As expected, the pressure at the front side of the cylinders outweighs and makes the fluid velocity on the cylinder surface approach the zero which indicates the fluid stagnation point. Also, due to the presence of cylinders, pressure at the rear of the last cylinder decreases which indicates lower drag force in this region. As can be seen from Figure 8, dimensionless fluid velocity against time at the rear of the last cylinder would experience much more fall as opposed to rest of the cylinders.

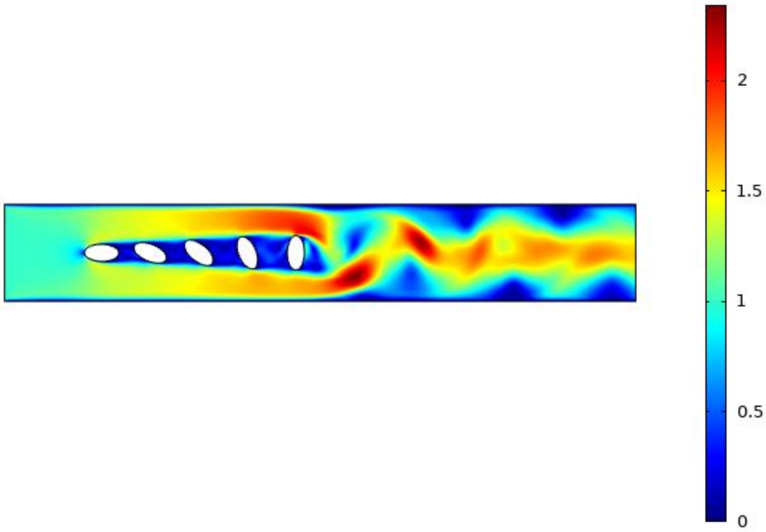
t*=31.47



t*=31.53



t*=31.6



t*=31.66

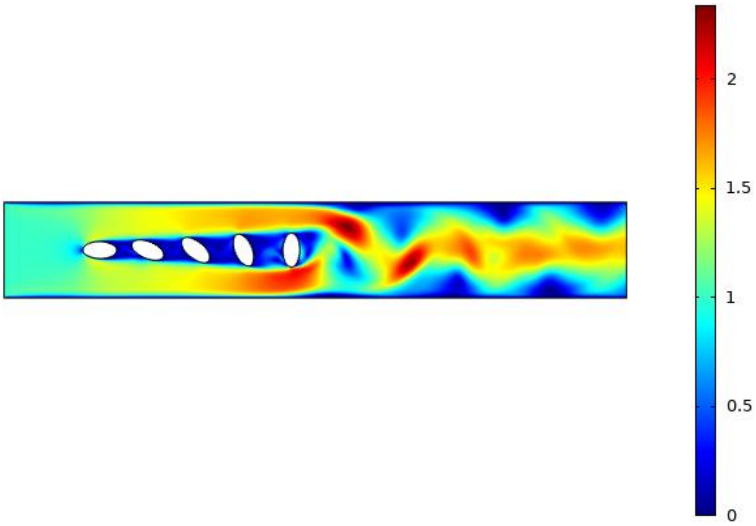


Figure 8. Dimensionless fluid velocity against dimensionless time.

Formation and release of the vortices, which cause vortex shedding in streets and influence the downstream flow of the elliptical cylinders, are obvious in Figure 8. To have a better understanding of the flow nature, the streamlines against dimensionless time are obtained in Figure 9. According to the streamlines, as the number of cylinders increases, the number of vortices and their magnitudes grow due to the presence of obstacles against the flow, and the biggest vortices will appear at the rear of the last cylinder. By moving away from the cylinders in the Y direction, the features of the boundary layer disappear so that the velocity field and pressure correspond with the free stream, as perceived by the contours presented in Figure 9.

It is also noticeable that the magnitude of the generated vortices increases due to the arrangement of cylinders rotating along the channel length and increasing the angle of cylinders one by one, creating larger obstacles in the flow direction. An increase in the Reynolds number as a result of increasing the free stream velocity can further decrease the pressure at the front side of the cylinder, consequently leading to a further decrease in the drag force coefficient as well as an increase in the vortex length. According to the contours of the streamlines at different dimensionless times, it can be concluded that the magnitude of vortices, particularly in the regions between cylinders, increases, indicating disorders and further turbulence across the flow field. The dispersion of vortices is due to the low Reynolds number and low velocity field, so if this is the case, the vortices are easily dispersed.

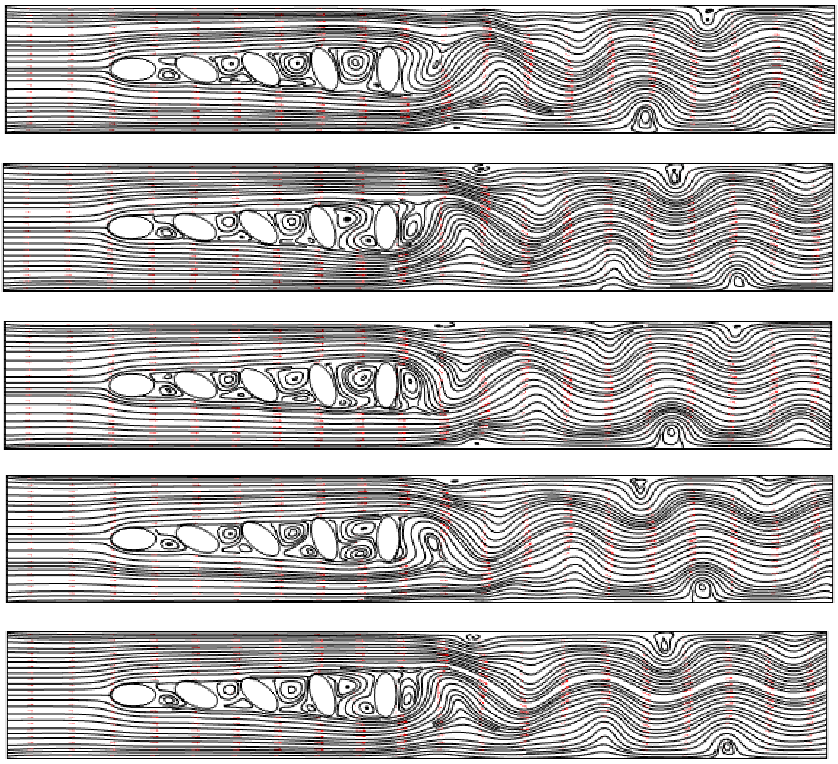


Figure 9. Contour of stream lines for each dimensionless time.

In addition to the downstream flow over the cylinders, the vortices generated between the cylinders are also evident. In fact, the magnitude and shape of these vortices vary over time, leading to significant variations in the flow field and heat transfer rate. To clarify this, the contour of the dimensionless temperature at different dimensionless times in a periodic fluctuation is presented in Figure 10. The free stream flow with a low temperature enters the channel and thermally interacts with the cylinders, causing its temperature to rise. With regard to the interaction between the fluid and the cylinders, only a trace of heat remains in the middle of the channel. These traces of heat start from coordinate 4 and last until the outlet, which is located at point 11. Also, as seen from Figure 10, these traces are stronger at the inlet section, while they fade towards the outlet section. At the rear of Cylinder number 5, which is angled at 90 degrees, the heat flood appears symmetrically. According to Figure 10, since the velocity field between the cylinders is lower than the free stream, the fluid's interaction with the cylinders intensifies, leading to an increase in the heat transfer rate. Hence, an increase in temperature in the regions between the cylinders is observed, although the amount of this increase varies along the channel length due to changes in the cylinder's angle and fluid deviation.

t*=31.4

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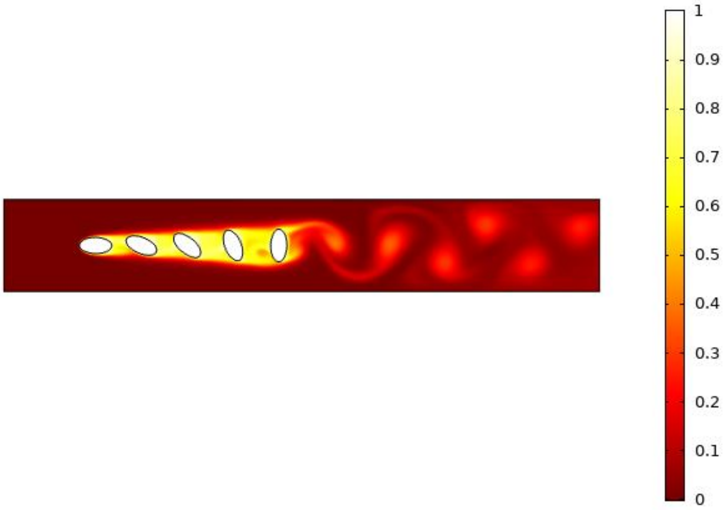
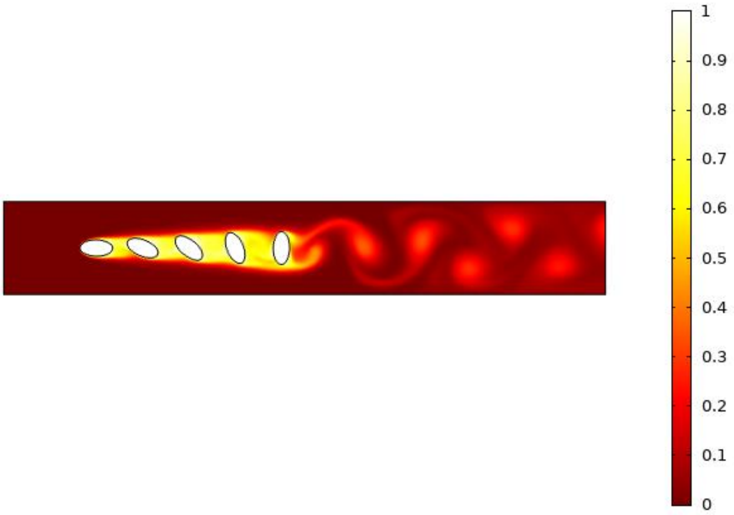
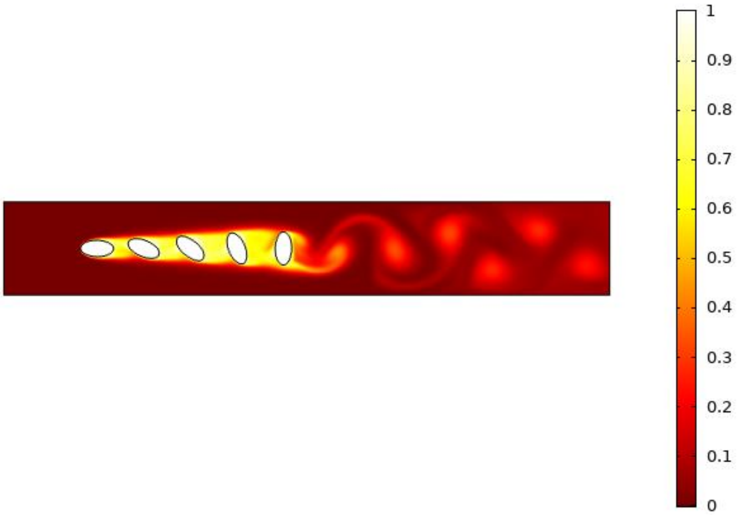


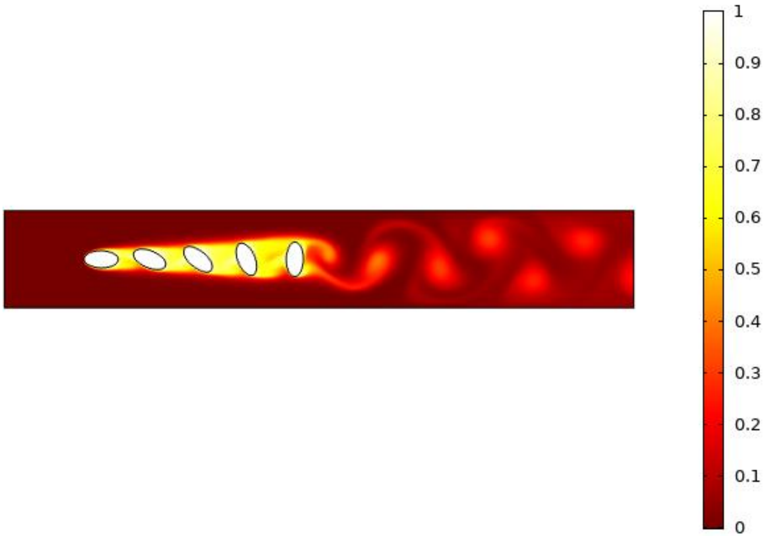
t*=31.46



t*=31.53



t*=31.6



t*=3.67

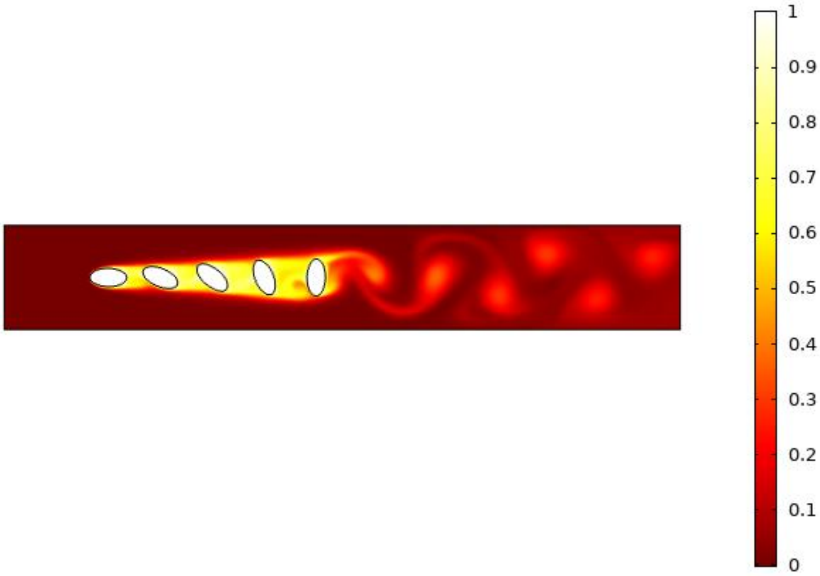
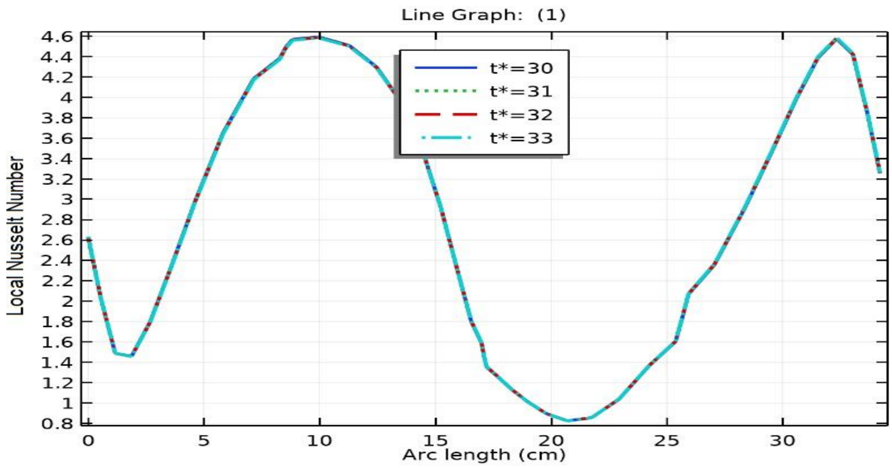
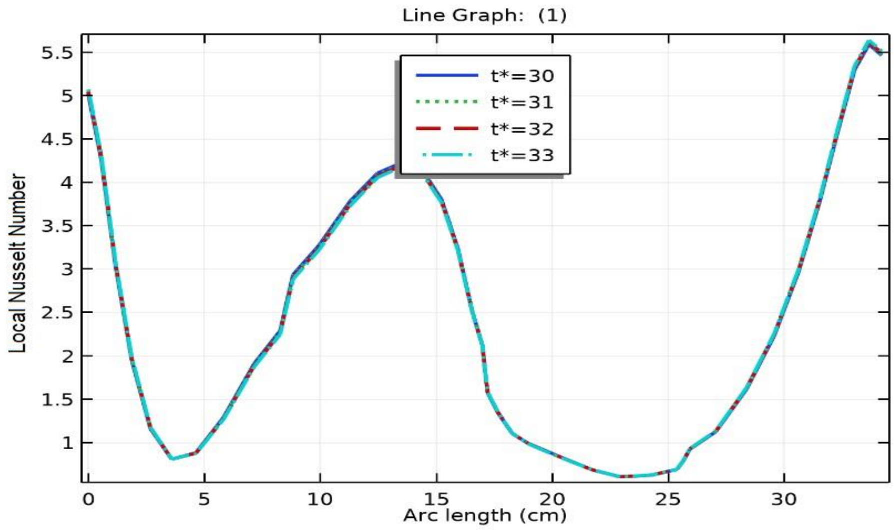
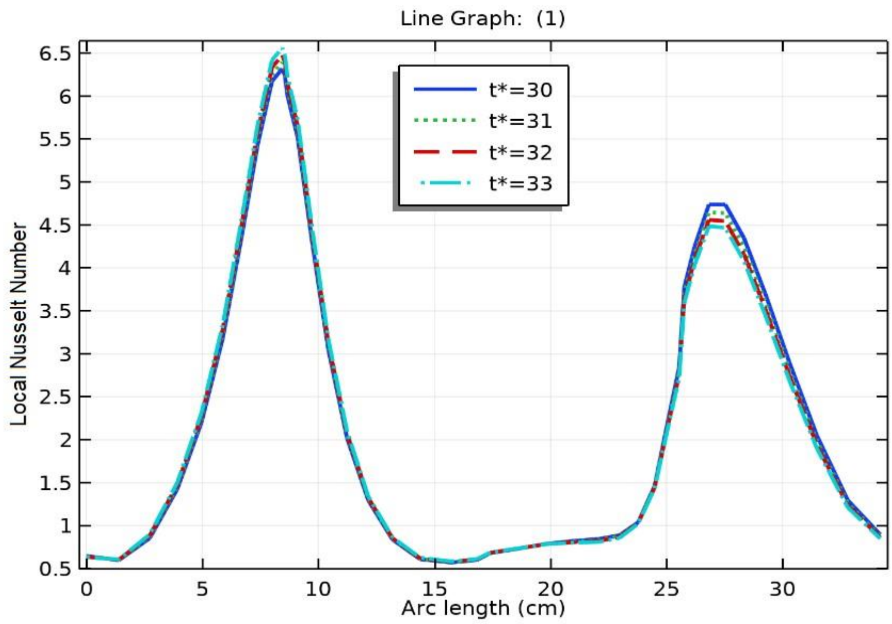


Figure 10. Contour of dimensionless temperature against dimensionless time.

By investigating the contour of dimensionless temperature, it is concluded that flow instabilities affect the heat distribution around the cylinders as well as temperature of downstream flow. In this regard, local Nusselt number variation against the dimensionless time is depicted in Figure 11.


Line Graph: (1)
Local Nusselt Number
Arc length (cm)
t*=30
t*=31
t*=32
t*=33


Line Graph: (1)
Local Nusselt Number
Arc length (cm)
t*=30
t*=31
t*=32
t*=33


Line Graph: (1)
Local Nusselt Number
Arc length (cm)
t*=30
t*=31
t*=32
t*=33

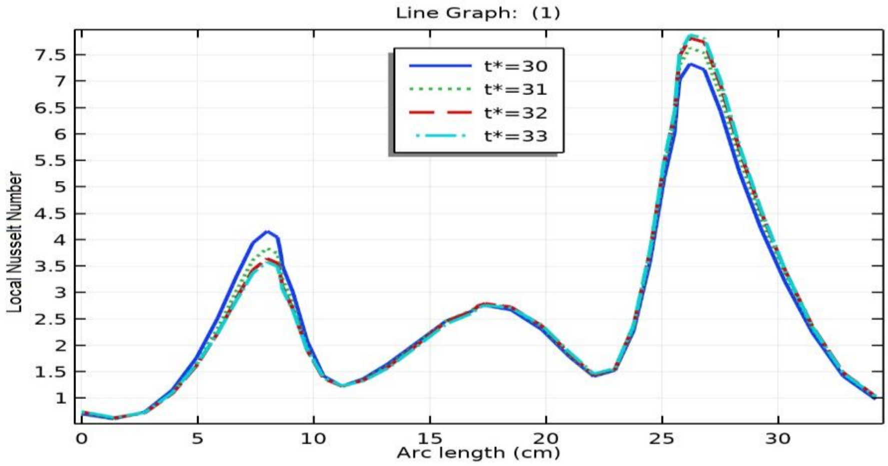


Figure 11. Local Nusselt number variation against dimensionless time.

As mentioned above, dimensionless time accounts for the influence of fluid flow velocity and system geometry on the heat transfer process. At different points in time, the flow and temperature distributions around the rotating elliptical cylinders change, leading to variations in transient heat transfer characteristics. It also plays a significant role in determining the stability and periodicity of fluid flow over the cylinder bundle. According to Figure 11, the local Nusselt number variation with time for the first cylinder is the least while it starts increasing by increasing the cylinder's angle in a way that the Nusselt number variation for cylinder number 5 is maximum. During the early stages of flow over the bundle of rotating elliptical cylinders, the Nusselt number has indicated a transient behavior that is influenced by factors such as the initial conditions, flow velocity, and the properties of the fluid itself. It can be seen from Figure 11 that the local Nusselt number rises rapidly due to the flow rapidly adapting to the presence of cylinders. Aside from dimensionless time, the aspect ratio of the rotating elliptical cylinders also plays a crucial role in determining the behavior of the local Nusselt number. More to the point, it can be concluded from Figure 10 and Figure 11 that the aspect ratio of elliptical cylinders has a significant impact on transient convective heat transfer. In fact, the results shed light on the significance of the aspect ratio and eccentricity of the individual cylinders when analyzing the overall transient heat transfer process. According to the results, the rotation of elliptical cylinders promotes the formation of vortices and enhances transient convective heat transfer. This increased heat transfer can be attributed to the generation of secondary flows and the improved mixing of fluid near the cylinder surface. Additionally, the aspect ratio and eccentricity of the cylinders play a crucial role in influencing the transient heat transfer rate. Higher aspect ratios lead to greater vortex shedding and stronger heat transfer, while higher eccentricities contribute to the asymmetrical distribution of vortices, thereby impacting the transient convective heat transfer process.

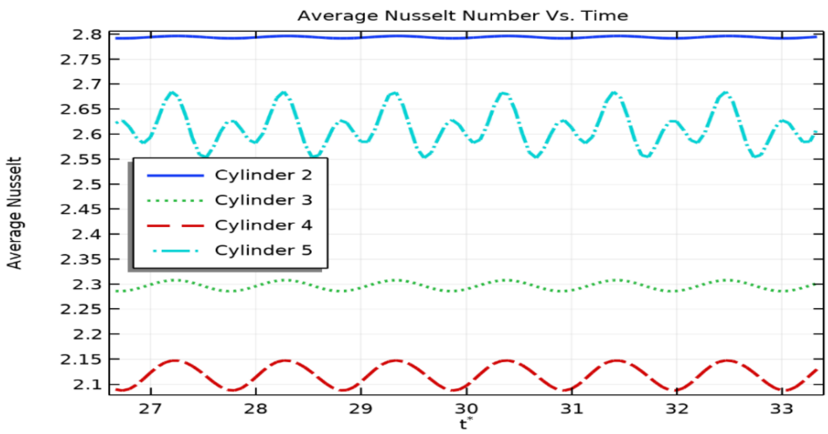


Figure 12. Mean Nusselt number variation for each single cylinder against dimensionless time.

To calculate the Strouhal number for investigating the lift coefficient behavior against dimensionless time, the Fourier analysis method is used to determine the vortex formation frequency. Figure 13 indicates the lift coefficient variation against dimensionless time for each cylinder. Generally, the behavior of the lift coefficient for all cylinders is periodic against time. As can be seen from Figure 13, the least amount of lift coefficient is allocated to cylinder number 2 owing to an inconsistent phenomenon in fluid dynamics. Also, it is concluded that cylinder number 5 possesses the maximum amount of vortex shedding frequency as well as the lift coefficient variation domain so that it endures the maximum drag force. In contrast, cylinder number 2 has shown to have the minimum vortex shedding frequency as well as the least lift coefficient variation domain. It is also reported that the amount of vortex shedding in the channel along with elliptical cylinders linearly increases by increasing the number of cylinders because the back cylinders are positioned in the low-pressure area in comparison with the front side cylinders.

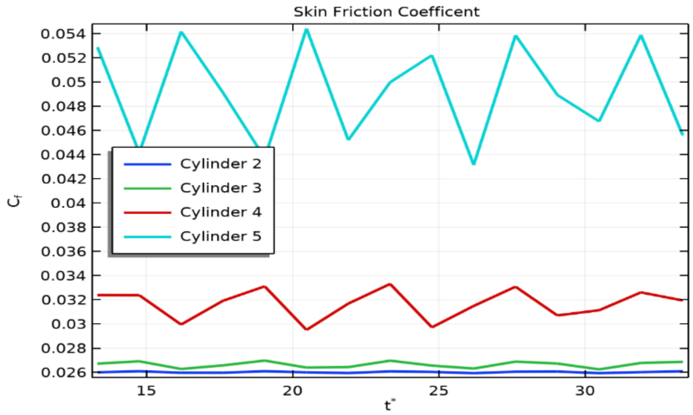


Figure 13. Lift coefficient variation against dimensionless time.

The interaction of the fluid with the cylinders results in the generation of vortices at the back of each cylinder, which disappear in the upstream region. As the fluid pressure at the back of the cylinder decreases, more vortices will be generated in the region. On the other hand, the adverse pressure gradient is reported to be the least due to the specific design of the cylinders, which are gradually inclined, and the shape of the cylinders, which allows the fluid pressure to be distributed over a wider area. This causes a delay in boundary layer separation, and as a result, a lower pressure drop against the flow occurs. The variation in mean pressure drop at the inlet section of the channel over time is shown in Figure 14. According to Figure 14, the pressure drop across the channel varies with time, as do other variables.

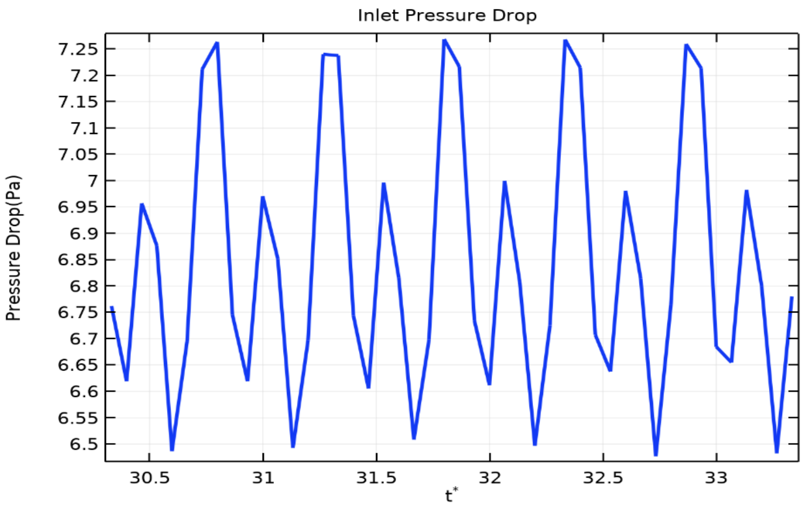


Figure 14. Pressure drop variation in the channel inlet section against dimensionless time.

6. Conclusion

In this paper, flow over the elliptical cylinders within a rectangular channel was investigated. The arrangement of cylinder inside the channel was designed in a way that the cylinders in the flow direction are rotated under a certain pattern. Here, 5 inclined cylinders whose angle gradually varies from zero to 90 degrees were considered. The studied parameters were Nusselt number, velocity field, vortex shedding which all is considered to vary against dimensionless time. By using numerical calculations of flow field and heat transfer around the cylinders, it was concluded that the generated vortexes could retain the symmetry at both sides of the cylinders. According to the results, the heat transfer amount around the cylinders increased due to cylinder's rotation. Also, contour of stream lines indicated that the maximum drag force and vortexes generated are imposed on the last cylinder owing to upstream obstacles. While, it was seen that the least drag force and vortexes are associated with the cylinder number 2. In terms of pressure drop, it was reported that rotating cylinders cause minimum pressure drop in the flow direction. Also, it was revealed that arrangement of inclined cylinders along the channel would not cause unsymmetrical vortexes.

References

1. Mangrulkar, C.K., et al., *Thermal performance escalation of cross flow heat exchanger using in-line elliptical tubes.* Experimental Heat Transfer, 2020. **33**(7): p. 587-612.
2. Kumar, R. and N.K. Singh, *Large eddy simulation of flow over elliptic cylinder array in square configuration at subcritical reynolds numbers.* Journal of Thermal Engineering, 2021. **7**(1): p. 204-219.
3. Abolfathi, S., et al., *Experimental study on flow around a tube in mixed tube bundles comprising cam-shaped and circular cylinders in in-line arrangement.* International Journal of Thermal Sciences, 2021. **163**: p. 106812.
4. Sakr, R.Y., et al., *Experimental investigation of heat transfer from elliptic tubes heat exchanger immersed in a fluidized bed.* Heat and Mass Transfer, 2022. **58**(12): p. 2047-2065.
5. Kumar, D. and A.K. Sahu, *Non-Newtonian fluid flow over a rotating elliptic cylinder in laminar flow regime.* European Journal of Mechanics-B/Fluids, 2022. **93**: p. 117-136.
6. Zhang, N., et al., *Fluid flow and heat transfer characteristics over a superelliptic cylinder at incidence.* Powder technology, 2020. **360**: p. 193-208.
7. Park, Y.G., et al., *Effect of the aspect ratio of an elliptical cylinder on mixed convective heat transfer within a lid-driven enclosure.* Journal of Mechanical Science and Technology, 2020. **34**: p. 3057-3068.
8. Vijay, K., et al., *Flow-induced transverse vibration of an elliptical cylinder with different aspect ratios.* Ocean Engineering, 2020. **214**: p. 107831.
9. Karlson, M., B.G. Nita, and A. Vaidya, *Numerical Computations of Vortex Formation Length in Flow Past an Elliptical Cylinder.* Fluids, 2020. **5**(3): p. 157.
10. Nag, P., M.M. Molla, and M.A. Hossain. *Non-Newtonian shear thinning effect on natural convection flow over an isothermal elliptical cylinder*. in *AIP Conference Proceedings*. 2019. AIP Publishing LLC.
11. Seo, Y.M., M.Y. Ha, and Y.G. Park, *A numerical study on the three-dimensional natural convection with a cylinder in a long rectangular enclosure. Part I: Size effect of a circular cylinder or an elliptical cylinder.* International Journal of Heat and Mass Transfer, 2019. **134**: p. 420-436.
12. Shi, X., et al., *Flow three-dimensionality of wavy elliptic cylinder: vortex shedding bifurcation.* Ocean Engineering, 2024. **301**: p. 117527.
13. Shi, X., et al., *Wake of wavy elliptic cylinder at a low Reynolds number: wavelength effect.* Journal of Fluid Mechanics, 2023. **969**: p. A22.
14. Kukreti, S. and N.K. Singh, *Aspect ratio effect on flow past an elliptical cylinder near a moving wall.* Physics of Fluids, 2023. **35**(6).
15. Duong, V.D., et al., *Near-moving-wall flows past three tandem elliptical cylinders at low Reynolds number of 150.* Physics of Fluids, 2024. **36**(1).
16. Li, Y., et al., *Numerical study on the flow around an elliptic cylinder with different aspect ratios in the subcritical regime using large eddy simulation.* Physics of Fluids, 2024. **36**(1).
17. Wu, Y., et al., *Flow-induced vibration of an elliptic cylinder with a splitter-plate attachment at low-Reynolds number: Self-limited oscillations.* Journal of Fluids and Structures, 2023. **122**: p. 103985.

18. Gupta, P., et al., *Effect of confinement on flow around a rotating elliptic cylinder in laminar flow regime.* Journal of Fluids Engineering, 2023. **145**(10).
19. Kumar, D. and A. Sahu, *Heat Transfer Characteristics of a Rotating Elliptical Cylinder to Power-Law Fluid in Laminar Flow Regime.* Available at SSRN 4643151.
20. Taloub, D., et al., *Numerical simulation of the natural double-diffusive convection in an elliptical cylinder-Impact of the buoyancy force.* International Communications in Heat and Mass Transfer, 2023. **144**: p. 106790.
21. Derakhshandeh, J.F. and N. Gharib, *Numerical studies of laminar flow over two tandem elliptical cylinders using Ramanujan approximation.* Journal of the Brazilian Society of Mechanical Sciences and Engineering, 2021. **43**(3): p. 169.
22. Zhu, J. and L.E. Holmedal, *A numerical study of separation and stagnation points for steady and unsteady flow over an elliptic cylinder near a moving wall.* Physics of Fluids, 2021. **33**(8).
23. Masliyah, J.H. and N. Epstein, *Steady symmetric flow past elliptical cylinders.* Industrial & Engineering Chemistry Fundamentals, 1971. **10**(2): p. 293-299.
24. Matos, R., et al., *Three-dimensional optimization of staggered finned circular and elliptic tubes in forced convection.* International Journal of Thermal Sciences, 2004. **43**(5): p. 477-487.
25. Alawadhi, E.M., *Laminar forced convection flow past an in-line elliptical cylinder array with inclination.* Journal of heat transfer, 2010. **132**(7).
26. Žukauskas, A., *Heat transfer from tubes in crossflow*, in *Advances in heat transfer*. 1972, Elsevier. p. 93-160.
27. Knudsen, J.G., D.L. Katz, and R.E. Street, *Fluid dynamics and heat transfer.* Physics Today, 1959. **12**(3): p. 40.
28. Churchill, S. and M. Bernstein, *A correlating equation for forced convection from gases and liquids to a circular cylinder in crossflow.* 1977.
29. Roshko, A., *On the drag and shedding frequency of two-dimensional bluff bodies*. 1954.
30. Norberg, C., *Fluctuating lift on a circular cylinder: review and new measurements.* Journal of Fluids and Structures, 2003. **17**(1): p. 57-96.
31. Williamson, C.H., *Defining a universal and continuous Strouhal–Reynolds number relationship for the laminar vortex shedding of a circular cylinder.* The Physics of fluids, 1988. **31**(10): p. 2742-2744.
32. Meneghini, J., et al., *Numerical simulation of flow interference between two circular cylinders in tandem and side-by-side arrangements.* Journal of fluids and structures, 2001. **15**(2): p. 327-350.
33. Braza, M., P. Chassaing, and H.H. Minh, *Numerical study and physical analysis of the pressure and velocity fields in the near wake of a circular cylinder.* Journal of fluid mechanics, 1986. **165**: p. 79-130.
34. Ding, H., et al., *Numerical simulation of flows around two circular cylinders by mesh-free least square-based finite difference methods.* International journal for numerical methods in fluids, 2007. **53**(2): p. 305-332.