

Enhancement the DOA for 2D Coprime Array Using CNN

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Abstract

Direction of arrival is the most important issue in the array signal processing the use of traditional technique for the DOA estimation would require large number of array sensor this lead to unfeasible and in the same time very costly .so the use of coprime array will use less number of array with the same result of the use of traditional method the cooperation of this method with the deep learning method that suggested in this paper will provide efficient DOA estimation .the suggested scenario is to use 2D coprime array with convolutional neural network (CNN) to analyze the properties of the covariance matrixes of the incoming signals. The dataset use depends on the signal detected by the sensor this approach in the simulation show demonstration on the traditional method such as MUSIC and ESPRIT with error only 0.04 or the arrival angle.

Keywords: DOA estimation, MUSIC, deep learning, 2D coprime arrays, convolutional neural networks

I.Introduction

In wireless communication , acoustic imaging and radar system application the estimation of the direction of arrival is a critical issue in the signal processing [1]. In Conventional techniques to estimate the direction of Arrival depend on the use of beamforming and array processing[2]. In this method must understand the array shape and presuppose astatic environment [1]. However ,the increase of the use of deep learning methods in growth in DOA [2]. Deep learning has shown the potential to transcend the limitations of traditional techniques by gaining knowledge of the relationship between received signals and relevant angles of arrival [3]. This method eliminates the need to explicitly represent the matrix shape and can accommodate dynamic situations [2]. Additionally, deep learning enables the creation of sophisticated, adaptable techniques and the ability to accurately capture subtle connections within data, leading to greater differentiation in a specialized direction. [3]. Deep learning is ideal for big data problems, allowing for accurate direction of arrival predictions. Coprime arrays, with their low density, can be more accurate than traditional uniform linear arrays. 2D coprime arrays with deep learning offer advantages such as better interference resistance, ability to handle non-linearities in data, and better source identification. End-to-end training can also be performed, enabling reliable direction of arrival results in various applications[2]. Applying deep learning techniques in the estimation of the Direction of Arrival for 2D coprime arrays improves accuracy, increases the method's ability to handle noise and non-linearity, and allows for thorough training of the model for both estimates and future tasks [4]. The use of deep learning techniques for estimating the direction of arrival in 2D coprime arrays offers several advantages over traditional methods[5]. Some benefits of this system include improved accuracy, resistance to interference and non-linear deviations, and the ability to completely train the system for tasks like source localization or tracking. [6]. Furthermore, the combination of deep learning with 2D coprime arrays enhances the resolution and estimate of proximate sources. These applications include radar imaging, wireless communications, and sound signal processing. In conclusion, using deep learning methods to determine the arrival direction of signals in 2D coprime arrays has considerable potential in achieving very precise and dependable outcomes in angle estimation. Deep learning approaches integrated into the estimate of angles of arrival for 2D coprime arrays

provide significant improvements compared to conventional methods[2]. The benefits of this approach include enhanced precision, resilience to interference and complex relationships, and the capability to conduct comprehensive training for simultaneous estimate and future tasks.

Current DNN-based methods for DOA estimation utilize the independence of signals or the prior knowledge of the number of sources [9]. This paper presents a deep learning-based scheme for reliable recovery of the spatial spectrum, which is suitable for applications where the number of signals can be unstable. The proposed deep learning-based method is compared to the traditional system, the MUSIC algorithm. Simulations using CA indicate the suggested method can give a considerable reduction in MAE and significant flexibility in training settings. The main contributions of our work can be briefly depicted in the following:

- The proposed method resolves up to $\min(N^2, M^2)$ sources using $M^2 + N^2$ sensors.
- The proposed method uses deep learning for DOA estimation using CPA.
- The suggested approach attains accurate high-resolution Direction of Arrival (DOA) estimation using a limited number of sensors.
- The proposed method is robust to noise.

The paper has been divided into five parts. Section 2 covers the relevant mathematical foundations related to the issue. Section 3 discusses the modeling of Convolutional Neural Networks for Direction of Arrival estimation. Section 4 includes comparisons and simulations. Section 5 offers a summary of the complete paper.

II. Preliminaries

A. Signal Model for 2D Coprime Planar Array

The conventional 2D-coprime planar array (2D-CPA) consists of two uniform square subarrays as shown in figure 1. The first subarrays have $M \times M$ elements with inter-elements spacing Nd and the second subarray have $N \times N$ elements with inter-elements spacing Md . The total number of the elements in 2D-CPA is $L = M^2 + N^2 - 1$ since the two subarrays shared the reference element at $(0,0)$. The CPA elements are positioned at:

$$\mathbb{P}_1 = \{(k_1Md, k_2Md) \mid 0 \leq k_1, k_2 \leq N - 1\} \quad (1)$$

$$\mathbb{P}_2 = \{(k_3Nd, k_4Nd) \mid 0 \leq k_3, k_4 \leq M - 1\} \quad (2)$$

$$\mathbb{P}_{2D} = \mathbb{P}_1 \cup \mathbb{P}_2 \quad (3)$$

The DCA set of the two subarrays is expressed as follows:

$$\mathbb{D}_{2D} = \mathbb{D}_{self} \cup \mathbb{D}_{cross} = (\mathbb{P}_1 - \mathbb{P}_1) \cup (\mathbb{P}_2 - \mathbb{P}_2) \cup (\mathbb{P}_1 - \mathbb{P}_2) \cup (\mathbb{P}_2 - \mathbb{P}_1) \quad (4)$$

The contiguous lag of DCA is limited due to the presence of holes that reduce the number of U DOFs, especially the critical holes that affect the range of the contiguous lag that can be represented as follows [10]:

$$\mathbb{H}_1 = \{(N, gM) \mid 1 \leq g \leq G, G = \lfloor N/M \rfloor + 1\} \quad (5)$$

$$\mathbb{H}_2 = \{(gM, N) \mid 1 \leq g \leq G, G = \lfloor N/M \rfloor + 1\} \quad (6)$$

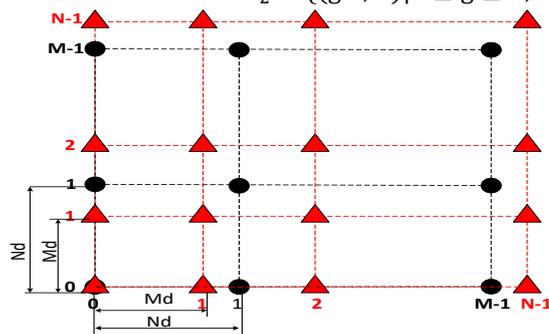


Figure 1: 2D-CPA structure

B. DOA Estimation of 2D-CPA

Suppose there is Q far-field narrowband uncorrelated sources are impinging in the antenna array with power $\{\sigma_1^2, \sigma_2^2, \dots, \sigma_q^2\}$ from directions $\{(\theta_q, \phi_q) | q = 1, 2, \dots, Q\}$, where θ_q, ϕ_q is the elevation and azimuth angles of the q -signal respectively. The received signal at the i th elements is expressed as follows [11]:

$$x_i(t) = \mathbf{A}_i s(t) + n_i(t), i = 1, 2 \quad (7)$$

Where

$$\mathbf{A}_i = [a_{yi}(v_1) \otimes a_{xi}(u), \dots, a_{yi}(v_q) \otimes a_{xi}(u_q)] \quad (8)$$

Where $a_{xi}(u_q)$ and $a_{yi}(u_q)$ are the steering matrix of the x -axis and y -axis of the i th subarray which can denoted as [11]:

$$a_{xi}(u_q) = [1, e^{j2\pi d_{xi}(\sin \theta_q \cos \phi_q)}, \dots, e^{j2\pi (N-1)d_{xi}(\sin \theta_q \cos \phi_q)}] \quad (9)$$

$$a_{yi}(u_q) = [1, e^{j2\pi d_{yi}(\sin \theta_q \sin \phi_q)}, \dots, e^{j2\pi (M-1)d_{yi}(\sin \theta_q \sin \phi_q)}] \quad (10)$$

The covariance matrix is calculated by $R_i = E\{x_i x_i^H\}$, Vectorizing R_i , yields

$$\mathbf{z} = \text{vec}(R_i) = (\mathbf{A}_i^* \odot \mathbf{A}_i) s + \sigma^2 \mathbf{I}_i \quad (11)$$

\odot represents the Khatri–Rao product. As the (jk, i) th entry in $\mathbf{A}_i^* \odot \mathbf{A}_i$ has the form

$e^{j2\pi (p_{xk} - p_{yj})d_{xi}(\sin \theta_q \cos \phi_q)}$, \mathbf{z} can be regarded as an equivalent signal vector received from a virtual DCA

$\mathbb{D} = p_{xk} - p_{yj}$, with sensors located at the difference lags.

C. 2D-Improved Coprime Planar Array Configuration (2D-Improved CPA)

Considering the location of the critical holes in eqs.(3) and (4) that interrupt the ULA segment of the DCA, a 2D-ICPA is proposed to fill the holes by relocating some elements position without adding extra elements. The proposed 2D-ICPA configuration focuses on filling the holes that locate inside the range $\{|(x, y)| - (M + N) < x, y < (M + N)\}$. Since the DCA is symmetry around the origin, when the holes in the 1st quarter and 2nd quarter are filled, the holes in the 3rd and 4th quarter are also filled.

For the proposed 2D-ICPA as shown in figure2, the elements are positioned as follows:

$$\mathbb{P}_1 = \{(k_1 Md, k_2 Md) | 0 \leq k_1, k_2 \leq N - 2\} \quad (12)$$

$$\mathbb{P}_2 = \{(k_3 Nd, k_4 Nd) | 0 \leq k_3, k_4 \leq M - 1\} \quad (13)$$

$$\mathbb{P}_3 = \{(aNd, bMd) | 1 \leq a \leq 2, 1 \leq b \leq \lfloor N/M \rfloor + 1\} \quad (14)$$

$$\mathbb{P}_4 = \{(bMd, aNd) | 1 \leq a \leq 2, 1 \leq b \leq \lfloor N/M \rfloor + 1\} \quad (15)$$

$$\mathbb{P} = \mathbb{P}_1 \cup \mathbb{P}_2 \cup \mathbb{P}_3 \cup \mathbb{P}_4 \quad (16)$$

The top and right elements of the 2D-CPA of N -subarray that locate in the positions $\{(M(N - 1)d, nMd); (nMd, M(N - 1)d) | 0 \leq n \leq N - 1\}$ are moved to the new location.

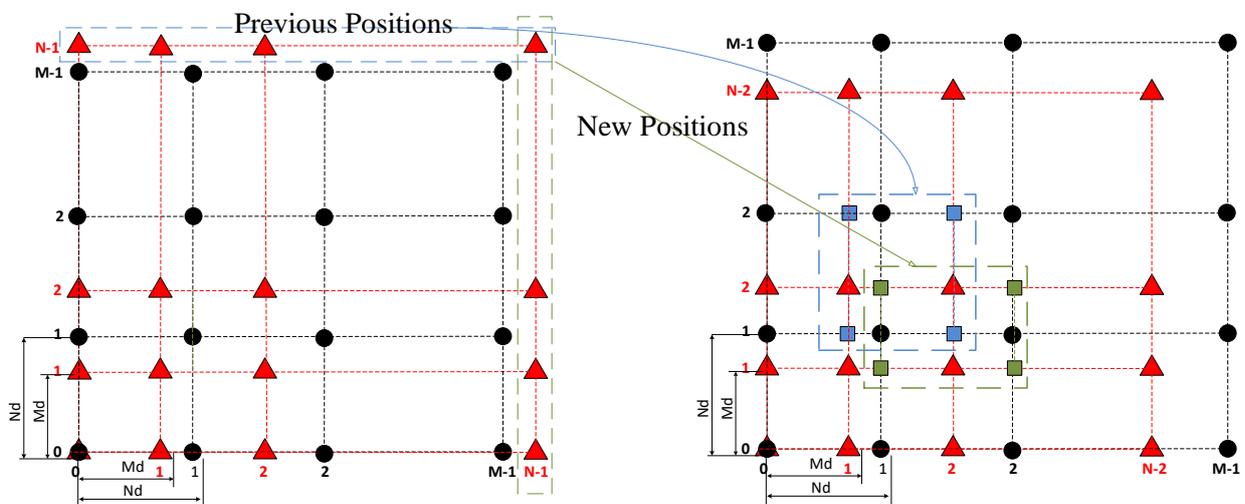


Figure 2: 2D-ICPA configuration

To illustrate the 2D-ICPA array performance in term of the number of the DOFs that can be generated, the analysis of the DCA and new element position is performed in the following subsections.

D. Relocation of Elements in 2D-ICPA for Holes Filling

The proposed 2D-ICPM is based on repositioning the removed elements from N-subarray to fill the critical holes and new generated holes in \mathbb{D} that locate inside the range $\{(x,y) | -(M+N) < x,y < (M+N)\}$ to generate a contiguous lag in such range. Regarding the critical holes given by eq. (1) and eq. (2) and for $N > M$, the new elements are position at (aN, bM) and (bM, aN) since these positions are not reside in the actual 2D-CPA. The holes can be filled from the lag constructed from the difference between the new relocated elements and the elements existed in the original array that located at $(n_0N, n_0N), n_0 = [0,1]$ and $(m_0M, m_1M), m_0 = [0,1,2], m_1 = [0,1]$.

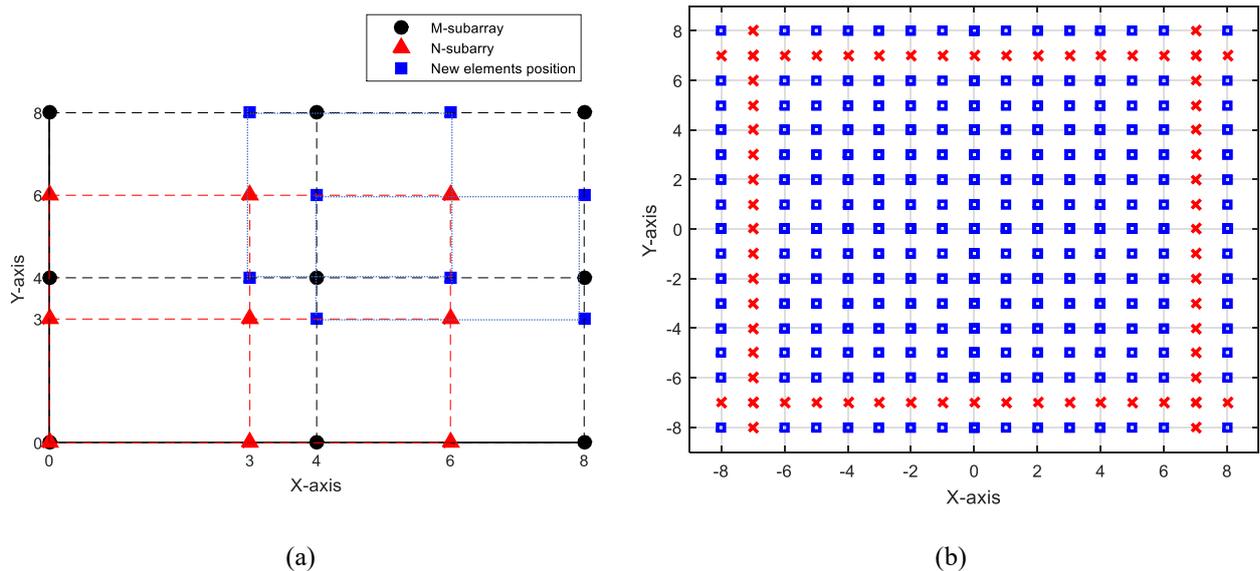


Figure 3: Example of 2D-ICPA with M=3, N=4 (a) element position (b) DCA

III. Deep learning suggested model

First the preprocessing of the data before entering to the deep learning model the data we use is the covariance matrix in 11 that drive from the conventional method this matrix is asymmetrical matrix the upper triangle has the same value of the lower triangle so it can be use any one of them ,the values of the matrix is complex and the deep learning model can't handle complex value so the suggestion is to use two matrixes one for real value and the other is for the imaginary and concatenate the amplitude and the phase at each instance the data is normalized before it use for the DNN that suggested the data is split into testing and training the testing is 33% of the total data with learning rate of 0.001 ,batch size 20 and dropout rate of 0.025. this is the parameter for our model

The suggested model consists of 10-layer except the input layer and the output layer as following

$$f(\mathbf{X}) = f_9 \left(f_7 \left(\dots f_1(\mathbf{X}) \right) \right) = \mathbf{z} \quad (25)$$

The architecture of the 1st layer of CNN network is consisting of 1D convulsion and padding using RELU as non-linear activation function where $\text{ReLU}(x) = \max(0, x)$,max pooling layer to filter the data the 3rd layer is drop out layer for the 2nd 1D convulsion layer consist also as the first layer followed by max pooling and Flatten layer followed by dense layer also use RELU as its activation layer ,dropout layer ,dense layer . For the last layer use dense layer without padding and with soft max as the activation function to estimate 9 non coherent angle the total epoch of this model is 150. The number of non-coherent source that the model can be detect is 9 sources.

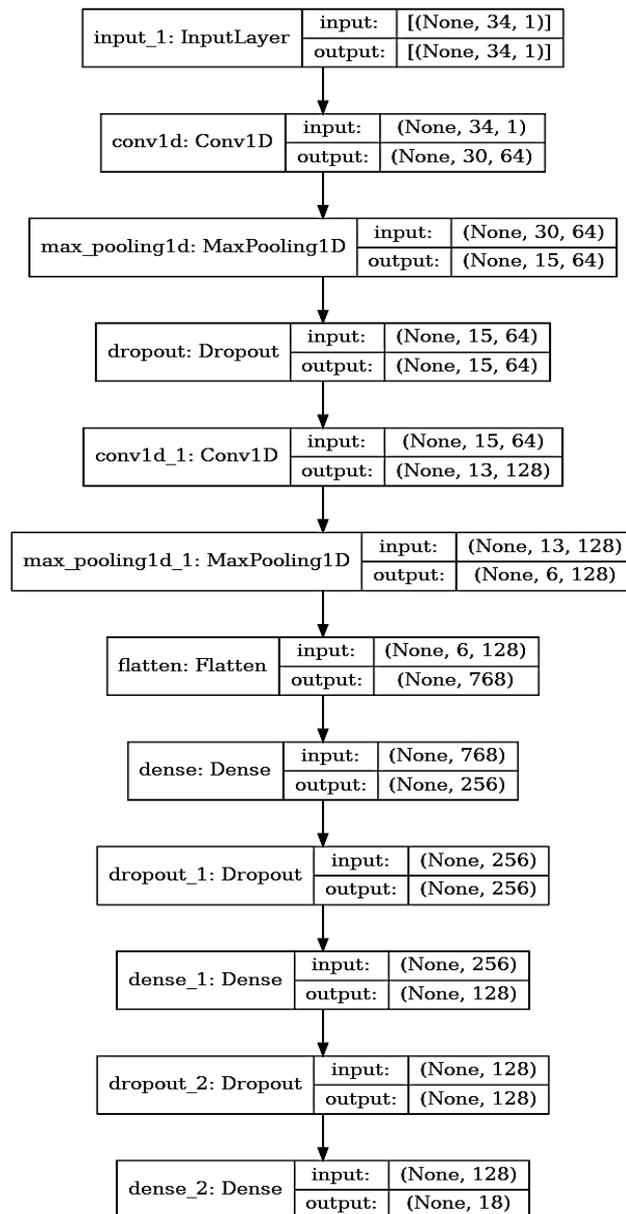


Figure 5: DL Suggested Model

IV.Result

In this part, we provide comprehensive simulation results to assess the effectiveness of the proposed Convolutional Neural Network (CNN) in estimating the Direction of Arrival (DOA) in different scenarios. Initially, we provide a concise overview of the techniques with which the suggested Convolutional Neural Network (CNN) was contrasted. In this evaluation, we assess the CNN's effectiveness on the assumption that the quantity of sources is already determined.

As an example, for illustrating the proposed 2D-Improved CPA when $M = 3, N = 4$, figure 3 shows the proposed array configuration with its DCA. The resulted DCA has a central URA with 169 contiguous lags located with the range $[-6: 1: 6] \times [-6: 1: 6]$, which improved the number of DOFs as compared to 2D-CPA in figure 2.

The suggested simulation scenario has 9 incoherent signals with the directions from -60 to 60 with $\Delta = 1^\circ$, for the azimuth and from -90 to 90 for the elevation with SNR=0dB, center frequency 2GHz and 450 snapshots. The spectrum estimation of the scenario shows in Figure 6.

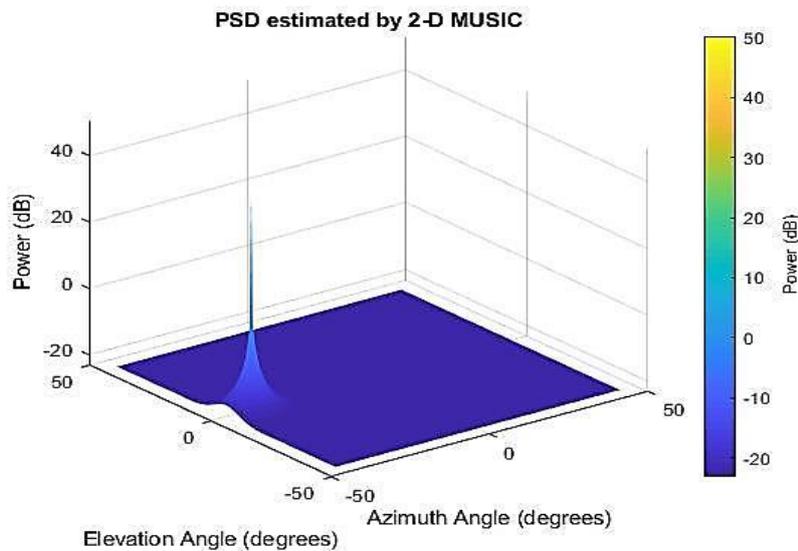


Figure 6: PSD estimation using MUSIC for the 2D coprime array

Using the data model, the training and testing data used for simulation the research models are generated (11). Next, the neural network underwent training to identify the occurrence of signals in designated spatial domains. The training process included the use of pairs consisting of calculated correlation vectors and outputs from the detection network. In order to determine the DOA signal k , a loss function is used that is based on the idea of mean square error (MSE).

$$\begin{aligned} \text{loss} &= \mathbb{E} \left\{ \|\theta_k - \hat{\theta}_k\|^2 \right\} \\ &= \frac{1}{DN} \sum_{m=1}^M \sum_{k=1}^D \|\theta_k - \hat{\theta}_k\|^2, \end{aligned} \quad (26)$$

Here, N is denoted as the number of examples, while $\hat{\theta}_k$ is noted as the prediction. Thereafter, based on the loss function as Eq. (26)

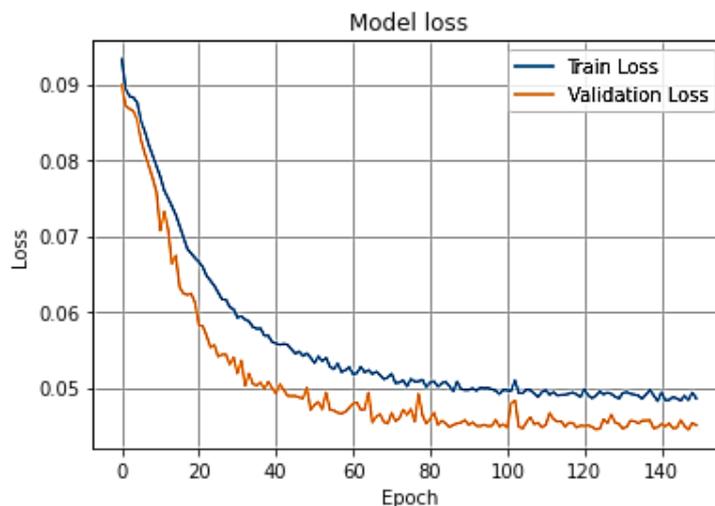


Figure 7: The Loss and Epoch of The Model

To evaluate the accuracy of the proposed deep learning-based direction of arrival (DOA) estimation technique, we use the well-recognized mean squared error (MSE) concept to measure the estimate error. The MSE is calculated as follows:

$$\text{MSE}_d = \frac{1}{M} \sum_{m=1}^M \|\theta_k - \hat{\theta}_k\|^2 \quad (27)$$

The result of MSE is shown in fig 8 for the designed model the average MSE is 0.04506

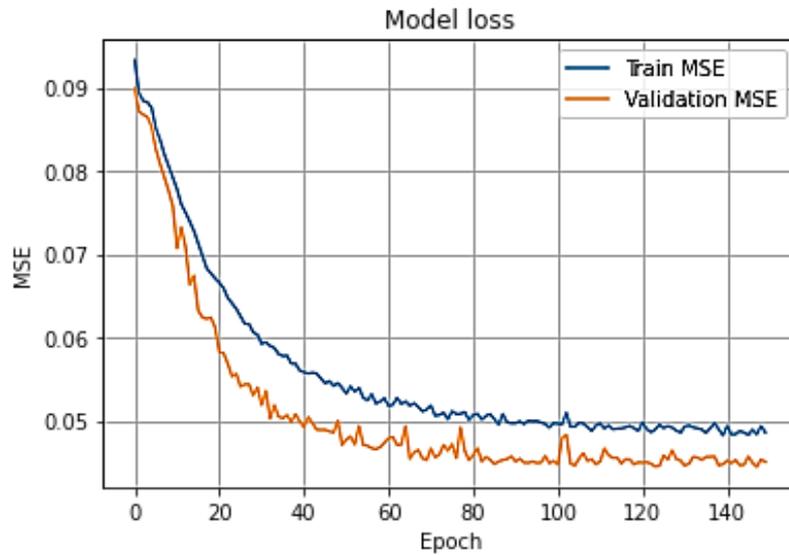
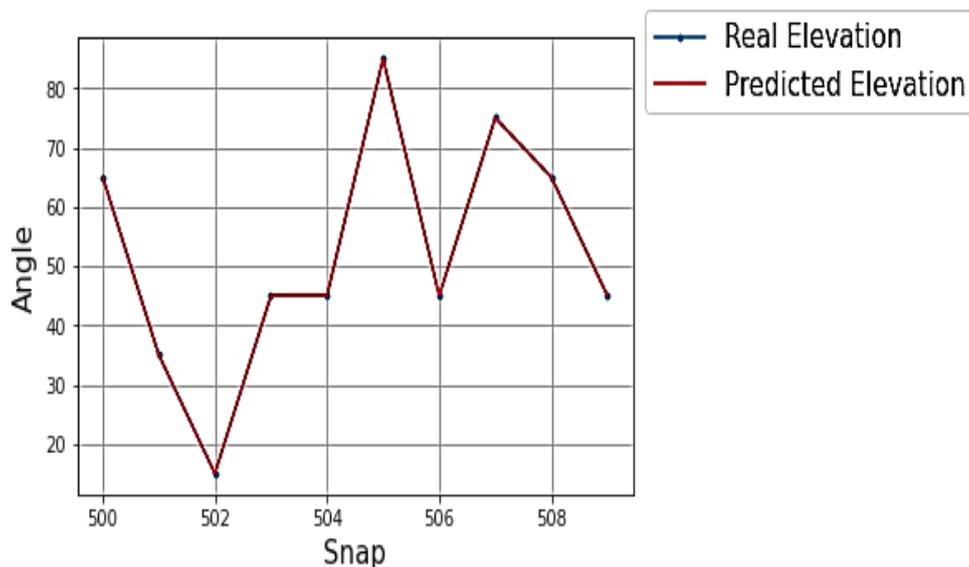


Figure 8: MSE loss according to epoch

In fig9 show the estimation of the elevation angle for each instance for range of snap shot of 500 to510 snap and compare the real with the predicted angle the accuracy of the suggested model is 99.54 and the time it take to estimate the DOA for each signal is 2ms



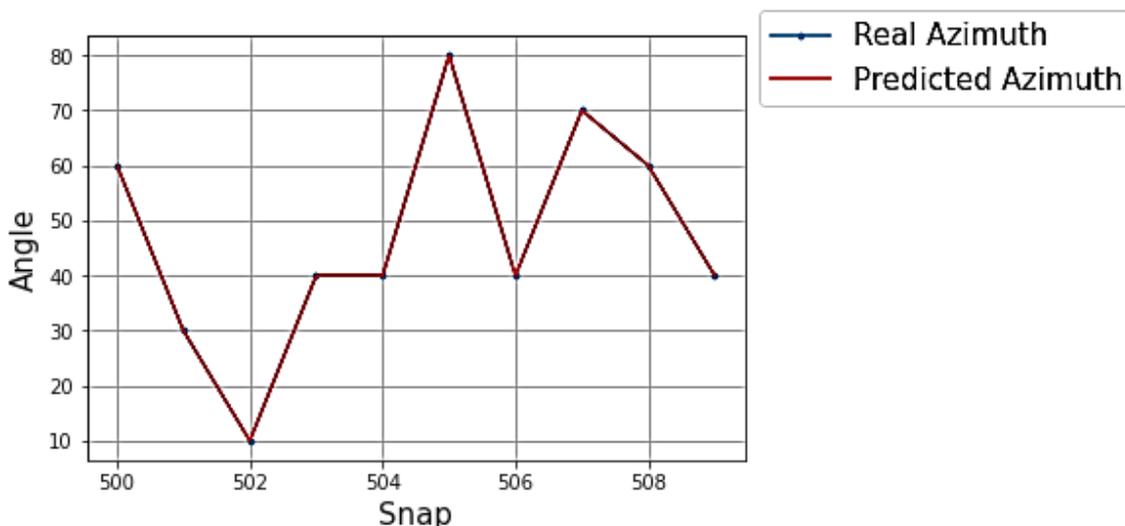


Figure 9: the estimation of azimuth angle and elevation angle using deep learning for viruses' snapshot

In order to assess the influence of noise on the neural network's performance, we compared the Root Mean Square Error (RMSE) of DOA estimates obtained from four different algorithms: the conventional MUSIC and our novel DOA estimation algorithm employing DNN. The RMSE, a measure of accuracy, was used to quantify the differences between estimated DOA values and actual DOA values. It is defined as follows:

$$RMSE = \sqrt{\frac{1}{QN_c} \sum_{q=1}^Q \sum_{n_c=1}^{N_c} (\theta'_q(N_c) - \theta_q)^2}$$

Where: θ'_q is the actual value of DOA and is the n_c Monte Carlo trial's projected DOA. specifies the number of Monte-Carlo trials, which has been set to 450. For the proposed technique, forward spatial smoothing, and forward/backward spatial smoothing, RMSE with regard to SNR and snapshots is compared. For RMSE with regard to SNR, the SNR value range is between -8 dB and 8 dB, there are 9 incoherent signals, and there are 450 snapshots.

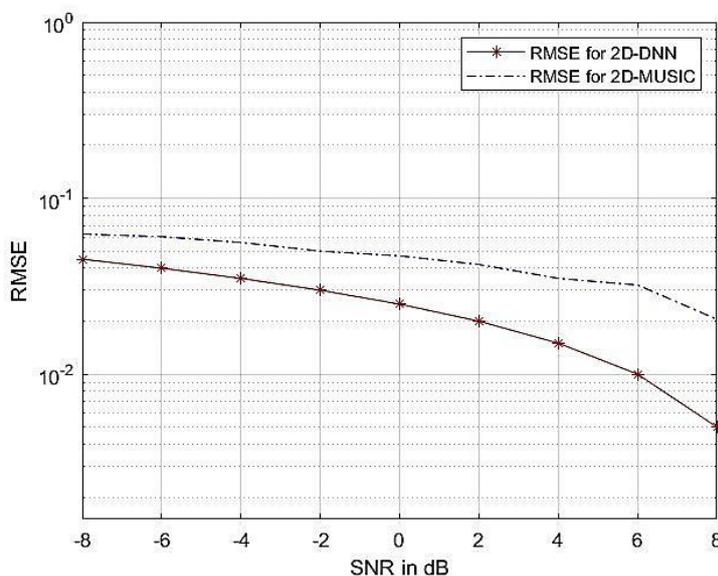


Figure 10: Root Mean Square Error Comparison

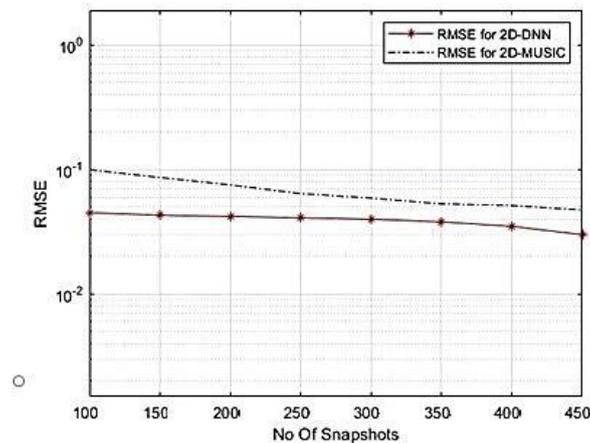


Figure11 : RMSE with the NO of snapshot

V. Conclusion

The proposed scheme use of CNN to implement feature extraction from the covariance matrix of the received signal. Their training is over simulated coprime array signals, and thus the method generalizes well. Simulation results show that the proposed method improves performance remarkably compared to conventional methods of DOA such as MUSIC and ESPRIT. The proposed method is noise- and clutter-insensitive.

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