Numerical investigation of natural convection in an inclined wavy solar collector containing a nanofluid

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ABSTRACT

The present paper gives a numerical study of natural convection heat transfer inside the inclined wavy solar collectors containing nanofluid in Bechar region. Bechar is located in the southwest of Algeria and belongs to the semi-arid zone. The solar deposit of this city corresponds to an angle of inclination of α =37 . The collector has a wavy absorber and cover. The inclined corrugated walls are maintained at constant temperature but different values. The vertical walls are assumed to be adiabatic. The nanofluid used is Al₂O₃-water. This problem consists in solving the system of equations containing: mass conservation, Navier Stokes and energy with adapted simplified hypotheses. Transport equations are solved numerically by finite element method. Governing parameters are taken as Rayleigh number (from 10 5 to 4×10^6), volume fraction (from 0 to 10%), amplitude of wavy cover (from 0 to 0.2) and wave of number (from 1.5 to 3.5). Results are presented by stream function and overage Nusselt numbers.

1. INTRODUCTION

The solar collectors which convert solar radiation to thermal energy then transfer it towards the carrying fluid has several practical and industrial applications, including electrical production, energy systems heating and cooling [1-3]. The solar collectors are designed usually as (flat) shallow enclosures but their absorbers can be different geometry such as wavy or corrugated. Also, they mounted with an inclination angle to receive more solar radiation [4]. However, the flat-plat collector has relatively low energy efficiency and low outlet temperatures. Recently, several studies have been carried out on the flat plate solar collector solar collector to overcome these disadvantages [2, 5], in addition to these classical studies or methods, which are used to increase the thermal efficiency of this kind of solar collector, one of the latest effective method is to replace the conventional working fluid by other fluids, which having higher thermal properties.

Recently researchers have become interested in the use of nanofluids in collectors [6]. A nanofluid is a fluid which contains nanometer-sized solid particles. Nanofluids were introduced by Choi [7] and they have been proven to provide efficient heat transfer compared to conventional fluids [8]. Usually, the nanoparticles that are used in nanofluid are made of metals, carbides, carbon nanotubes or oxides [9-13].

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The problem of the natural convection in a cavity differentially heated is studied numerically by several researchers [14-20]. Marina S. Astanina et al [21] studied numerically laminar natural convection in a square cavity having two centered adherent porous blocks filled with an alumina/water nanofluid under the effect of horizontal temperature gradient. The obtained results revealed the heat transfer enhancement at the hot wall with the Darcy number, while a growth of the porous layers size reduces the heat transfer rate at this hot wall. The behavior of the average Nusselt number at the right cold wall is opposite. Sheikholeslami et al. [22, 23] was examined natural convection heat transfer in a porous medium with nanofluid. They found that the improvement of the heat transfer coefficient is related to the increase of nanoparticles volume concentration.

Various studies have treated solar collectors with a corrugated and wavy absorber filled with a nanofluid. Yasin Varol et al. [24] presented a comparative numerical study on natural convection in inclined wavy and flat-plate solar collectors. It has observed that flow and thermal fields are affected by the shape of enclosure and heat transfer rate increases in the case of wavy enclosure than that of flat enclosure. R. Nasrin et al. [25] have analyzed numerically the effect of aspect ratio on convective flow inside a solar collector having the flat-plate cover and sinusoidal wavy absorber. The results showed that the performance of the collector can be improved by using the largest Aspect ratio of solar collector (Ar = L/H). The result of this study expresses a good agreement with the theoretical result available in the literature. M.M. Rahman et al. [26] has been studied a corrugated bottom triangular solar collector introducing water based nanofluids inside the enclosure. It has been found that both Grashof number and solid volume fraction have significant influence on streamlines and isotherms in the enclosure. It is also found that heat transfer increased by 24.28% from the heated surface as volume fraction φ increases from 0% to 10% at Gr = 106 and τ =1 for copper water nanofluid. Nasrin et al. [27] have showed a numerical study of forced convection heat transfer through a flat plate solar collector, the geometry is tested for 2D case, then 3D, where water and Cu-H₂O nanofluid are used as a working fluid. The results obtained show that 3D simulation is more efficient than 2D, and the increase of nanoparticles volume concentration gives an improvement in heat transfer rate reach 17% and about 8% for thermal efficiency of solar collector. Ammar Maouassi et al. [28] illustrated with numerical study of nanofluid (water-SiO₂) to stimulate solar flat plate collector efficiency with heat transfer modification properties. They concluded that heat transfer increases with increasing both nanoparticles concentration and Reynolds number.

In this paper, we investigate numerically the heat transfer of nanofluid (water-Al₂O₃) flows through an inclined wavy solar collector. We examined the effect of nanoparticles volume fraction, the Rayleigh number, amplitude of wavy cover and number of wave. The objective of this article is to introduce the convective heat transfer and the flux intensity of a corrugated solar collector used to exploit solar energy in our semi-arid region.

2. MATHEMATICAL MODEL

The inclined wavy solar collector geometry of the physical problem considered, and boundary conditions are shown in Figure 1. The nanofluid used in the collector is water-Al2O3 and is assumed to be Newtonian, isotropic and homogeneous. The flow is laminar, transient, incompressible and two-dimensional. The thermophysical properties of the nanofluid are assumed to be constant. The fluid phase and that of the nanoparticles are in a state of thermal equilibrium and flow with the same velocity. The solar collector is composed of a wavy cover plate on the top surface and a wavy absorber plate on the bottom. This corrugation is traced by the second-order Bezier curve. Both plates are maintained at fixed and determined temperatures. The length and height of the collector are L and H respectively. The density of the nanofluid is approximated by the Boussinesq model.

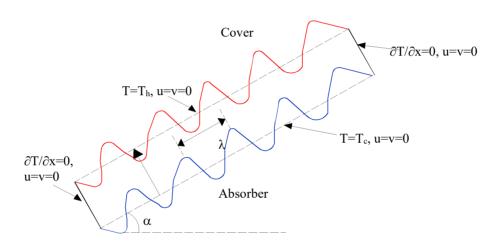


Figure 1: Physical model and boundary conditions.

The non-dimensional version of the governing system of transport equations are as follow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial x} + \frac{\mu_{eff}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \beta_{nf} \left(T - T_f \right) g \sin \left(\alpha \right)$$
(2)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial y} + \frac{\mu_{eff}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \beta_{nf} \left(T - T_f \right) g \cos(\alpha) \tag{3}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$
 (4)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$
 (5)

The thermal diffusivity, the effective density, the heat capacitance and the thermal expansion coefficient of nanofluid are given by:

$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}} \tag{6}$$

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \tag{7}$$

$$\left(\rho C_p\right)_{nf} = (1-\phi)\left(\rho C_p\right)_f + \phi\left(\rho C_p\right)_S \tag{8}$$

$$(\beta)_{nf} = (1 - \phi)(\beta)_f + \phi(\beta)_s \tag{9}$$

The original closed-type impeller wheel is shown in Figure 1 a), and the modified opentype impeller wheel calculated according to the methodology above is shown in Figure 1 b).

$$\phi \left(\frac{k_s - k_{eff}}{k_s - 2k_{eff}} \right) + (1 - \phi) \left(\frac{k_f - k_{eff}}{k_f + 2k_{eff}} \right) = 0$$
 (10)

$$\mu_{eff} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{11}$$

Based upon the previous assumptions and introducing the following dimensionless variables,

$$(x^*,y^*) = \frac{(x,y)}{H}, (u^*,v^*) = \frac{H(u,v)}{\alpha_f}, T^* = \frac{(T-T_c)}{(T_h-T_c)}, P^* = \frac{PH^2}{\rho_f \alpha_f^2},$$

The governing equations for the problem in dimensionless form are as follows:

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y^*} = 0 \tag{12}$$

$$\left(1 - \phi + \phi R_{\rho}\right) \left(\frac{\partial u^{*}}{\partial t^{*}} + u^{*} \frac{\partial u^{*}}{\partial x^{*}} + v^{*} \frac{\partial u^{*}}{\partial y^{*}}\right) = -\frac{\partial P^{*}}{\partial x^{*}} + \frac{\Pr}{\left(1 - \phi\right)^{2.5}} \left(\frac{\partial^{2} u^{*}}{\partial x^{*2}} + \frac{\partial^{2} u^{*}}{\partial y^{*2}}\right) + \Pr Ra \sin(\varphi) \left(1 - \phi + R_{\rho} R_{\beta} \phi\right) T^{*} \tag{13}$$

$$\left(1 - \phi + \phi R_{\rho}\right) \left(\frac{\partial u^{*}}{\partial t} + u^{*} \frac{\partial u^{*}}{\partial x} + v^{*} \frac{\partial u^{*}}{\partial y^{*}}\right) = -\frac{\partial P^{*}}{\partial x^{*}} + \frac{\Pr}{\left(1 - \phi\right)^{2.5}} \left(\frac{\partial^{2} u^{*}}{\partial x^{*2}} + \frac{\partial^{2} u^{*}}{\partial y^{*2}}\right) + \Pr Ra \cos(\varphi) \left(1 - \phi + R_{\rho} R_{\beta} \phi\right) T^{*} \tag{14}$$

$$\frac{\partial T}{\partial t}^* + u^* \frac{\partial T}{\partial x}^* + v^* \frac{\partial T}{\partial y}^* = \left[\frac{K_{nf}}{K_f} \right] \left(\frac{1}{(1-\phi) + \phi R_{\rho} R_{Cp}} \right) \left(\frac{\partial^2 T}{\partial x^{*2}} + \frac{\partial^2 T}{\partial y^{*2}} \right)$$
(15)

Non-dimensional parameters can be given as:

$$\Pr = \frac{\mu_{eff}}{\rho_f \alpha_f^2}, \quad Ra = \frac{\rho_f \beta_f H^3 g \Delta T}{\mu_f \alpha_f}, \quad R_\rho = \frac{\rho_s}{\rho_f}, \quad R_\beta = \frac{\beta_s}{\beta_f}, \quad R_k = \frac{K_s}{K_f}, R_{C_p} = \frac{C_{ps}}{C_{pf}}$$

The heat transfer is characterized by the flux ratio between nanofluid and fluid. The average flux ratio at the hot inclined corrugated wall is computed as follows:

$$\frac{Q_{nf}}{Q_f} = -\int_0^l \frac{K_{nf}}{K_f} \left(\frac{\partial T}{\partial x}^* \cos \alpha + \frac{\partial T}{\partial y}^* \sin \alpha \right)$$
 (16)

Dimensionless boundary conditions for Eqns. (11)-(14) are as follows:

- $u^* = v^* = 0$ and $T^* = 1$ at wavy cover surface
- $u^* = v^* = 0$ and $T^* = 0$ at wavy absorber surface
- $u^* = v^* = 0$ and $\frac{\partial T^*}{\partial x^*} = 0$ at vertical walls

3. NUMERICAL METHOD AND VALIDATION

The nonlinear governing partial differential equations, i.e., mass, momentum and energy conservation equations are transformed into a system of integral equations by using the Galerkin weighted residual method of finite-element formulation. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations with the help of Lagrange's method. Lastly, these linear equations are solved by applying Triangular factorization. In our work, several simulations are taken into account according to the amplitude values and the number of waves. The number of elements varies between 2048 to 9664.

To ensure the accuracy of the available code, the numerical results were compared with similar works. Consequently, a more general case, in which the natural convection is present in a wavy solar collector subjected to solar irradiance in top wall, was used for comparison purpose. A comparison is made between results of the average Nusselt number obtained from this study with Nesrin published results [25] as shown in Figure 1. The numerical solutions (present work and Nesrin et al. [25]) are in good agreement.

4. RESULTS AND DISCUSSION

In this numerical analyze, the average Nusselt number and maximal stream function has been carried out for various values of physical parameters such as amplitude of wave A , number of wave λ , nanoparticles volume fraction and Rayleigh number with an Al2O3/water nanofluid in a solar collector. The effect of the heat transfer is studied using the following range of values are given Rayleigh number ($105 \le Ra \le 4 \times 106$), volume fraction of nanoparticles from $\phi=0$ to $\phi=10\%$, amplitude of wavy cover (0, 0.025, 0.05, 0.1 and 0.2), number of wave from $\lambda=1$ to $\lambda=3.5$, inclination angle of cavity $\alpha=37^\circ$ and Prandtl number Pr=7. The thermophysical properties of the water base fluid and Al2O3 nanoparticles are shown in Table 1.

Table 1: Thermophysical properties of Al₂O₃-water [32]

Thermophysical properties	Al ₂ O ₃	Water	
ρ (kg.m ⁻³)	3970	997.1	
$C_p(J/kg.K)$	765	4179	
K(W/m K)	40	0.6	
β (1/K)	0.85×10^{-5}	21×10 ⁻⁵	

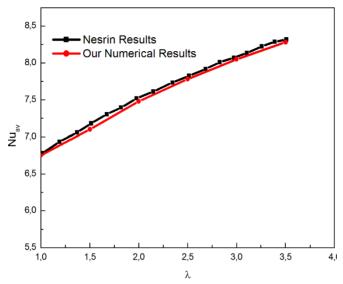


Figure 2: Comparison between present code and Nesrin results [25].

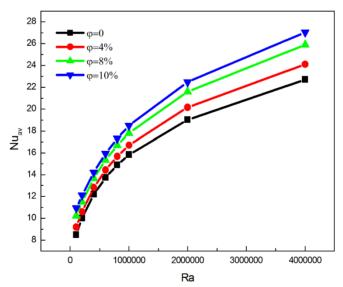


Figure 3: Variation of the average Nusselt number as a function of the Rayleigh number for different values of the nanoparticles volume fraction

The evolution of the average Nusselt number as a function of the Rayleigh number at different values of the volume fraction is illustrated in figure 3. The increase in Rayleigh number leads to an increase in the average Nusselt number, which means that the rate convection transfer is favored inside the solar collector. Also, the addition of nanoparticles improves the average Nusselt number compared to the base fluid (water). This improvement is justified by the large value of the thermal conductivity of the nanoparticles.

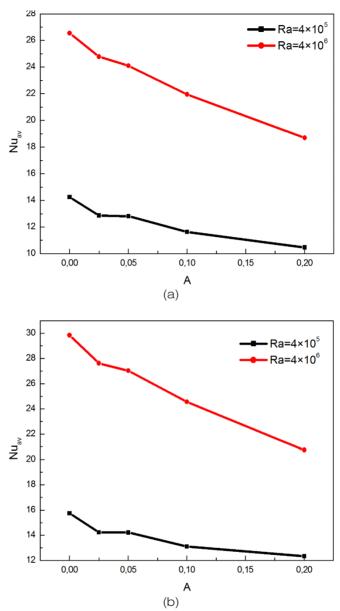


Figure 4: Variation of average Nusselt number depending on the wavy cover amplitude for two values of Rayleigh number for: (a) - φ =0.04 and (b) - φ =0.1.

Figure 4 shows the variation of Nusselt number as a function of the amplitude of the wavy cover for two values of Rayleigh number at ϕ =0.04 and ϕ =0.1. In figure (4.a), the average Nusselt number decreases with the increase in the amplitude of the wavy cover which means that the transfer rate by convection and the intensity of the flow decrease. The same remarks for figure (4.b) except that the average Nusselt number values get larger because of the increase in the volume fraction.

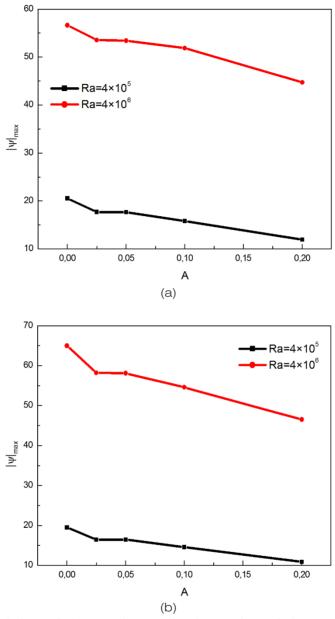


Figure 5: Variation of the maximum absolute value of the stream function depending on the wavy cover amplitude for two values of Rayleigh number for: (a) - ϕ =0.04 and (b) - ϕ =0.1.

Figure 5 presents the variation of the maximum stream function as a function of the amplitude of wavy cover for two Rayleigh number value at ϕ =0.04 and ϕ =0.1. The two figures confirm the previous observations.

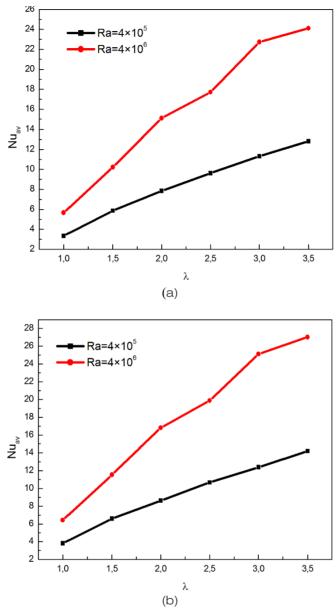


Figure 6: Variation of the average Nusselt number as a function of the waves number for two values of Rayleigh number for: (a) - ϕ =0.04 and (b) ϕ =0.1.

Figure 6 illustrates the effect of the number of waves on the average Nusselt number for two values of Rayleigh number at ϕ =0.04 and ϕ =0.1. For the both figures (6.a and 6.b), the average Nusselt number increases with the number of waves. At Ra=4×105 and for λ =2.5, Nu_{av}=9.6268 and it increases by a ratio of 27.7% for λ =3.5. For Ra=4×106, the ratio between Nu_{av}(λ =2.5) and Nu_{av}(λ =3.5) is 26.5%. Therefore, the increase in the number of undulations leads to a dominance of the convective mode.

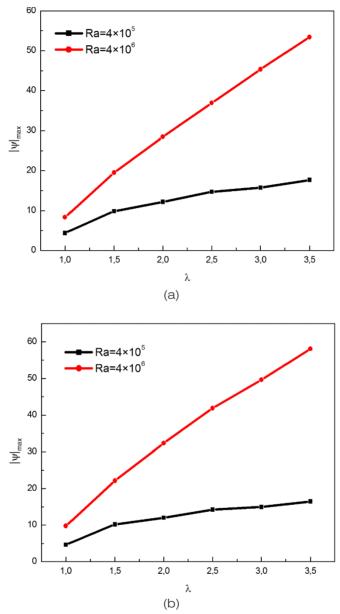


Figure 7: Variation of the maximum absolute value of the stream function depending on the waves number for two values of Rayleigh number to (a) - ϕ =0.04 and (b) - ϕ =0.1

Figures (7-a) and (7-b) represent the evolution of the maximum absolute value of the stream function depending on the waves number for two values of Rayleigh number for ϕ =4% and ϕ =10%, respectively. The two curves indicate that $|\psi|_{max}$ increase with increasing the cover wave number. This increase shows that the intensity of the flow and the force of circulation enhances with the waves number. This is owing to the increment in the viscosity forces.

5. CONCLUSIONS

In this paper, the flow and heat transfer characteristics of Al2O3 through a wavy solar collector, for Rayleigh number values from 105 to 4×106 , amplitude of wavy cover from 0.025 to 0.2, number of waves from 1 to 3.5, with a wide range of volume concentration (0 to 10%) are studied numerically. First, the presence of nanoparticles in the base fluid (pure water), increases significantly Nusselt number and maximum absolute value of the stream function, that is also proportional with the increase of Rayleigh number. Second, the amplitude of the waves decreases the Nusselt number and the maximum value of the current function. Third, the number of corrugations enhances the heat transfer by convection (increase in Nuav) and the intensity of the flow (increase in $|\psi|$ max). Finally, the Al2O3 nanoparticles with the highest wave amplitude and number of waves increase performance of heat transfer rate results in a good heat transfer between the energy systems compared to the base fluid (pure water) case.

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NOMENCLATURE

A	Amplitude height of the wavy wall
R_k	Ratio of thermal conductivities
g	Gravitational acceleration, m.s ⁻²
R_{β}	Ratio of thermal expansion coefficients

x, y Cartesian coordinates, m

P Pressure, N.m⁻²

P* Dimensionless pressure

 $\begin{array}{lll} Pr & Prandtl \ Number \\ Nu & Nusselt \ number \\ R_{\rho} & Density \ ratio \ [-] \\ Ra & Rayleigh \ number \\ T & Temperature \ [K] \end{array}$

t Time [s]

 x^*, y^* Dimensionless coordinates, m u, v Components of velocity fields, m.s⁻¹ R_k Ratio of thermal conductivities

u*,v* Dimensionless velocity components, m.s⁻¹

K Thermal conductivity, W m⁻¹.K⁻¹

GREEK SYMBOLS

α	Thermal diffusivity, m ² .s ⁻¹
β	Coefficient of thermal expansion, K ⁻¹
μ	Dynamic viscosity, kg m ⁻¹ .s ⁻¹
ρ	Density, kg.m ⁻³
$(\rho_{\rm C})$	Heat capacity, J m ⁻³ .K ⁻¹
φ	Volume fraction
Ψ	Stream function, m ² .s ⁻¹
λ	Number of waves

SUBSCRIPTS

f	Fluid properties	
nf	Nanofluid properties	
S	Solid properties	
c	Cold wall	
h	Hot wall	
eff	Effective	

* Dimensional properties

max Maximum av Average