

# An Improved Gurney Model to Predict Initial Velocity of Parallel-moving Rod-shaped Fragment

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## **ABSTRACT**

An improved Gurney model considering explosive energy waste originated in the process of driving rod-shaped fragment revolving, is proposed herein to predict initial velocity of parallel-moving rod-shaped fragment using parameters such as Gurney constant, mass ratio of explosive charge to rod-shaped fragments, and slanting angle of rod-shaped fragments arrangement. Static arena test was performed to evaluate validity of said improved model, and predictions based on the improved Gurney model exhibit impressive consistency with test results, providing a reliable methodology to predict initial velocity of parallel-moving rod-shaped fragments in the engineering development of controllable discrete rod warhead.

## **1. INTRODUCTION**

Amongst various warheads used in the anti-air and anti-missile weapon systems, controllable discrete rod warhead characteristic of rod-shaped fragment receives widespread attentions in industries for combined advantages of heavier rod-shaped fragment mass and higher fragment projecting velocity [1]. Controllable discrete rod warheads already in services comprise P-73E, improved P-73E of Russia Federation armed forces, and AIM-9L of the United States Air Force, etc.

Controllable discrete rod warhead has an inner explosive charge circumferentially surrounded by a series of rod-shaped fragments with identical slanting angle away from warhead axis. In the intercepting scenario explosive charge produces expansive explosion products, which drives rod-shaped fragments parallel-moving toward intended target, forming a set of rod-shaped fragments around warhead axis with 360 degree distribution pattern, while keeping a revolving motion around length of rod-shaped fragments. Finally, a complete ring of rod-shaped fragments is obtained within lethality range around warhead axis to cut, and destroy intended target [1].

Many scholars in industries had reached a consensus on prediction model calculating revolving speed around length of rod-shaped fragment [2-6], whereas keeping a disagreement on prediction model calculating initial velocity of parallel-moving rod-shaped fragments. ZHOU Difeng, HE Yong, etc, calculated initial velocity of parallel-moving rod-shaped fragments using basic Gurney model [7-8], producing undesirable deviation from arena test results for the reason of ignoring energy consumption of explosive charge

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resulting from revolving motion of rod-shaped fragment [8]. SUN Chuanjie, etc. attempted to improved basic Gurney model using a adjusting coefficient of  $\varepsilon$  [5], while without publishing detailed explanation of coefficient of  $\varepsilon$ .

In this paper an improved Gurney model considering explosive energy waste originated in the process of driving rod-shaped fragment revolving, is proposed herein to predict initial parallel-moving velocity of rod-shaped fragment. Static arena test was performed to evaluate validity of said improved model, and predictions based on the improved Gurney model exhibit impressive consistency with test results, providing a reliable engineering methodology to predict initial velocity of controllable revolving rod-shaped fragment in the course of warhead development program.

## 2. BASIC GURNEY MODEL

### 2.1 Gurney Assumptions

The Gurney equations are a set of mathematical formulas first developed in the 1940s by Ronald Gurney of the U. S. Army Ballistics Research Laboratory, which are extensively used in warhead engineering to predict how fast an explosive charge will accelerate surrounding metal fragments. Some assumptions were made as followed.

- 1) 100% energy conversion of explosive charge into kinetic energy of expansive explosion products and warhead case.
- 2) Expansion velocity of explosion products distributes linearly along warhead radius, and expansion velocity equals to moving velocity of fragments and warhead case.
- 3) Explosion products expasses with even pace after detonation of explosive charge, while keeping a identical distribution density of explosion products.
- 4) Edge effects of expansion explosive products are ignored.

### 2.2 Basic Gurney Equations

For general cylindrical explosive charge on the basis of Gurney assumptions and law of energy balance, we have:

$$CE = \frac{1}{2}Mv_0^2 + \frac{1}{2}\int_0^{a_f} v^2(r)2\pi r\rho(r)dr \quad (1)$$

Where  $C$  is specific mass of explosive charge along warhead cylinder.

$M$  is specific mass of warhead case along warhead cylinder.

$E$  is specific energy of explosive charge.

$v_0$  is initial velocity of fragment.

$v_0$  is fracturing radius of warhead case.

$v(r)$  is expassing velocity of explosive product at the distance of  $r$ .

$r$  is distance deviated from warhead axis.

$\rho(r)$  is density of expassive explosion products at the distance of  $r$ .

According to Gurney assumptions, equation (1) could be transformed into equation (2)[9]:

$$CE = \frac{1}{2}Mv_0^2 + \frac{1}{4}Cv_0^2 \quad (2)$$

Then Gurney equation (3) is obtained.

$$v_0 = \sqrt{2E} \sqrt{\frac{\beta}{1+0.5\beta}} \quad (3)$$

Where  $\sqrt{2E}$  is Gurney constant of given explosive charge, which could be measured in the experiments.

$M$  is metal mass along warhead cylinder.

$E$  is specific energy of explosive charge.

$v_0$  is initial velocity of fragment.

$\beta = C / M$  is mass ratio of explosive charge to fragments.

Based on basic Gurney equations within range of  $\beta = 0.1 \sim 5.0$ , velocity calculation of conventional fragmentation warhead shows impressive agreement with test results [9].

### 3 IMPROVED GURNEY MODEL

For controllable discrete rod warhead, chemistry energy released from explosive charge drives rod-shaped fragments parallel-moving while keeping revolving motion around length of rod-shaped fragment, according to Law of energy balance, equation (4) is obtained.

$$CE = \frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2 + \frac{1}{2}\int_0^{a_f} v^2(r) 2\pi r \rho(r) dr \quad (4)$$

Where  $\frac{1}{2}I\omega_0^2$  is revolving kinetic energy of rod-shaped fragments,

$I$  is moment of inertia of rod-shaped fragment, and  $I = ml^2 / 12$ ,

$m$  is weight of rod-shaped fragment,

$l$  is length of weight of rod-shaped fragment.

$\omega_0$  is initial revolving speed of rod-shaped fragment.

According to Gurney hypotheses, equation (4) could be transformed into equation (5):

$$CE = \frac{1}{2} M v_0^2 + \frac{1}{2} I \omega_0^2 + \frac{1}{4} C v_0^2 \quad (5)$$

On the assumptions that rod-shaped fragment having straight shape and even distributed mass density, while keeping a kind of structure with revolving axis perpendicular to center of rod-shaped fragment, initial parallel-moving velocity  $v_0$  has a relationship (6) with initial revolving speed  $\omega_0$  of rod-shaped fragment [2-4].

$$\frac{\omega_0}{v_0} = \frac{4}{l} \sqrt{\frac{1}{2} - \frac{1}{4\alpha} \sin(2\alpha)} \quad (6)$$

Where  $\alpha$  is slanting angle of rod-shaped fragments arrangement.

In the transformation of equation (4) and equation (5), moment of inertia  $I$  of rod-shaped fragments was wrongly described as  $I = ml^2 / 16$ , and adjustment should be made to correct equation (6), then an adjusted equation (7) is obtained.

$$\frac{\omega_0}{v_0} = \frac{2\sqrt{3}}{l} \sqrt{\frac{1}{2} - \frac{1}{4\alpha} \sin(2\alpha)} \quad (7)$$

Equation (5) and equation (7) are simultaneously solved, and equation (8) is obtained.

$$CE = \frac{1}{2} M v_0^2 + \frac{1}{2} \mu m v_0^2 + \frac{1}{4} C v_0^2 \quad (8)$$

Where  $\mu = \frac{1}{2} - \frac{1}{4\alpha} \sin(2\alpha)$ .

Finally, we get an improved Gurney equation (9).

$$v_0 = \sqrt{2E} \sqrt{\frac{C}{M + \mu m + 0.5C}} \quad (9)$$

When Equation (9) and equation (7) are simultaneously solved, initial revolving speed  $\omega_0$  of rod-shaped fragment could be obtained.

Equation (9) could be simplified as equation (10) when having a low mass ratio of warhead case to rod-shaped fragment.

$$v_0 = \sqrt{2E} \sqrt{\frac{\beta}{1 + \mu + 0.5\beta}} \quad (10)$$

According to equation (10), initial parallel-moving velocity  $v_0$  of rod-shaped fragment could be calculated with the help of such determined parameters as Gurney constant  $\sqrt{2E}$ , mass ratio  $\beta$  of rod-shaped fragments to explosive charge, and slanting angle of rod-shaped fragments arrangement  $\alpha$ .

## 4 EXPERIMENTS AND ANALYSIS

### 4.1 Experimental Results

In this study two experimental warheads are prepared to verify effectiveness of said improved Gurney model, specifications of experimental warhead show in Table 1.

Table 1: Specifications of Experimental Warheads

Sample series	#1	#2
Explosive charge	HMX-based PBX	RDX-based PBX
Mass of explosive charge (kg)	2.84	2.78
Mass of rod-shaped fragments (kg)	3.09	3.12
Length of rod-shaped fragments (mm)	171	171
Slanting angle of rod-shaped fragments arrangement (degree)	2	3

Experiment setup shows in Figure 1, target plates and velocity screens are arranged around experimental warhead axis at distances of 3 meters, 5 meters, and 7 meters respectively.

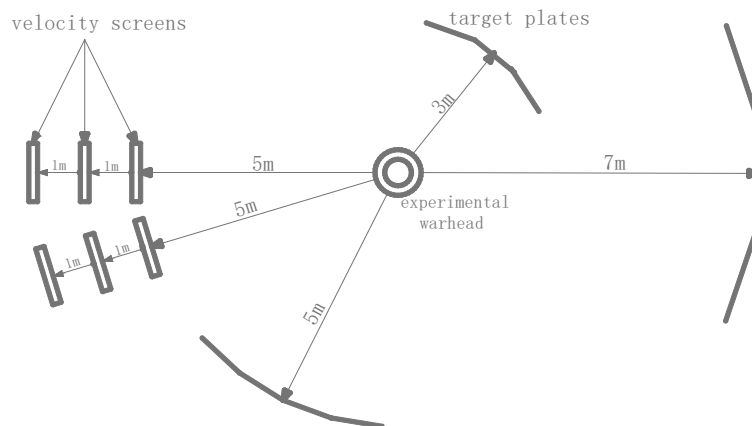
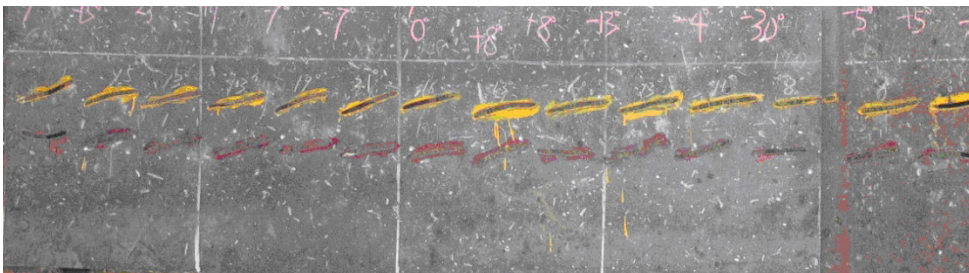


Figure 1: Experiment Setup



Figure 2: Velocity Screens Setup

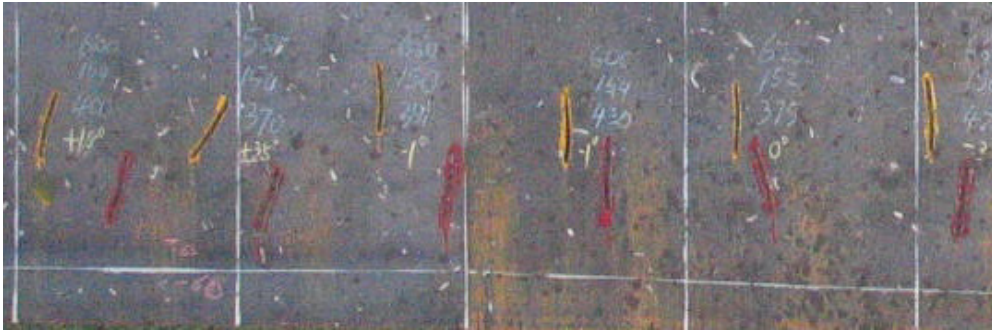


(a) Penetration of Target Plates at Distance of 3 Meters away from Warhead



(b) Penetration of Target Plates at Distance of 5 Meters away from Warhead





(c) Penetration of Target Plates at Distance of 7 Meters away from Warhead  
Figure 3 Penetration of Target Plates

As shown in Figure 3, parallel-moving rod-shaped fragments produce a circular ring of connecting penetration rectangular holes on the target plates, forming a “cutting ring” around experimental warhead axis capable of cutting intended target within lethality range.

#### 4.2 Analysis

On the basis of equation (10), we could calculate initial parallel-moving velocity  $v_0$  of rod-shaped fragments, and calculations and measurements are shown in Table 2, showing excellent consistency with measurements.

Table 2: Comparisons between Basic Gurney Predictions and Experimental Measurements

Sample	Measurements (m/s)	Predictions (m/s)	Deviation
#1	2004	2045	2.05%
#2	2039	1959	-3.92%

On the basis of basic Gurney equation (3), we could calculate initial parallel-moving velocity  $v_0$  of rod-shaped fragments, and calculations and measurements are shown in Table 3, exhibiting undesirable deviation from experimental measurements.

Table 3: Comparison between Improved Gurney Predictions and Experimental Measurements

Sample	Measurements (m/s)	Predictions (m/s)	Deviation
#1	2004	2365	18.01%
#2	2039	2268	11.2%

In Table 2, predicted velocity of initial parallel-moving rod-shaped fragments based on basic Gurney model deviates from measurements with no more than 3.92%, which is acceptable in the engineering development of discrete rod warhead.

In Table 3 predicted velocity of parallel-moving rod-shaped fragments based on improved Gurney model deviates from measurements with no less than 11.2%, which exhibits undesirable inaccuracy.

We draw a parallel between Table 2 and Table 3, and a conclusion is obtained that equation (10) of improved Gurney model could produce more accurate prediction of initial velocity of parallel-moving rod-shaped fragments than equation (3) of basic Gurney model.

## 5. CONCLUSIONS

- (1) Improved Gurney model should be obtained with consideration of revolving kinetic energy of rod-shaped fragments, which could be used to predict initial velocity of parallel-moving rod-shaped fragments using such parameters as Gurney constant, mass ratio of explosive charge to fragments, and slanting angle of rod-shaped fragments arrangement.
- (2) Two discrete rod warhead samples are prepared, and subject to static ground explosion experiment, results show a desirable consistency with improved predictions of Gurney model, providing a reliable engineering methodology to predict initial velocity of parallel-moving rod-shaped fragments.

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