

# Regulative Conditions of Existence of Doppler Effect Using Green Function Formalism

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## **ABSTRACT**

The Doppler shift of the near and far Waves due to the electromagnetic source in motion is investigated. Using the decomposition of Green's function for the wave in the case of classical oscillating electron motion, it is found that there are some regulative conditions of the phenomena. The present theory proves that the shift is undefined in the adjacent area. In fact, in the vicinity of the source the wave number is very small so that the delay effects become negligible and the phase is imprecise. Actually, the spectrum of the radiation field in the adjacent area is ambiguous. The current theory establishes a regular Doppler effect in the distant area. In fact, the wave number can never be achieved. Taking account of the source motion, the immobile observer can in all way stay sometime in the far field zone to obtain all the information on the field. Bearing in mind that the propagator coupled with the electric field takes a shifted frequency. Once, the relationship between the far-field frequency, the oscillating dynamic source have been established, the spectrum exists and can be analyzed..

## **1. INTRODUCTION**

Consider an observer in free space is immobile relative to a classical oscillating electron motion; the apparent color of the source is changed by its motion. The motion of the source causes the waves in the face of the source to be compressed whereas behind it to be extended. The electric field frequency measured by the observer will be upper than the source frequency. It tends to blue shift when approaching each other. Being lower, it tends to red shift particularly when it is more widely spaced. This effect is known as the Doppler effect [1]. As an invention it has mainly offered a subject for researchers with various applications ranging from the electromagnetic to acoustic domain [2]. The Doppler effect is essentially applicable to astronomy. It enables astronauts to calculate the speed of the light-emitting objects such as galaxies and Extra Solar Planets like Gliese 581c [3] was discovered in April 2007 by *Doppler* spectroscopy through the use of the HARPS spectrograph. This discovery is important in the domain of the free electron laser [4,5]. In fact, the oscillating electrons can be observed as an oscillating electric dipole. It offers radiation with the same frequency of oscillation, Doppler shifted when changed back to the laboratory frame. The most important result of such an effect is the dependence of the wavelength on the electron beam energy. Then, the FEL can be tunable over a large bandwidth.

As it is a known phenomenon in physics, the Doppler effect has been considered with diverse processes [6,7]. David Erzen simulating single charged particle motion in external magnetic and electric fields without take account the radiation created by the electron in

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case of the vicinity and far from the source [8]. The majority of these processes are simply involved in the way of how to determine the calculated frequency, regardless the electric field, despite it is recognized that the frequency of the electric field can not in any circumstances be measured prior to the finding of the field itself. Consequently, these processes can't explain, quantitatively, the physical method that causes classical oscillating electron motion which is expressed in the source of the electric field frequency. Since they do not address the electric field, these methods can't respond to the inquiry of knowing whether the Doppler effect is constantly regular or it is only applicable when some surroundings are fulfilled. The inquiry denotes that the present understanding about the Doppler effect is unfinished and a further research is needed. To implement this examination of Doppler effect, the dyadic Green's function for the electric field is useful. The standard dyadic Green function description of the electromagnetic field generated by an electric point source is modified and corrected by Keller [9]. Instead of dividing the field itself, Arnoldus [10] splits the dyadic Green's function through linking the field to its source. The result is conducive to any electric field whatever its source is. Different approaches are used to study the Doppler effect. First, the radiative term of Green's function which permits the calculation of the far electric field spectrum in integral form is only taken from the splitting. In this respect, the relationship between the far field frequency, the oscillating dynamic source is established. This relationship does not only clarify the physics in line with the Doppler effect quantitatively but also highlights the circumstances in which the effect is applicable. Second, the splitting is restricted to a quasi-static Green function where the retardation effects are entirely neglected.

In the current article, a general method is applied to examine the conditions in which the Doppler effect is appropriate. Maxwell's equations, in free space which means an absence of dielectric are combined, to form the second order electric vector wave equation. This equation is solved with the help of a matrix approach recognized as the dyadic Green's function. The Green's-function formalism grants a suitable straightforward method to calculate the electric field. The electric wave equation and the Green's-function method are introduced at first. Within this context, the electrons distributive charge is in motion, it is localized in space much smaller than the wavelength. Under these conditions, It is possible to develop, in space at a fixed time, the Dirac delta function in the Taylor series of the associated current density  $\mathbf{J}(\mathbf{r}_1, t)$  about the origin. In addition, in the vicinity of the source the wave number is very small so that the delay effects become negligible and the phase is imprecise. Thereafter, after some tedious algebra we demonstrate how this approach is used to show Doppler effect is regular only in far zone.

## 2. THE PHYSICAL PROBLEM AND CHOICE OF GREEN FUNCTION

Within the classical electromagnetic theory, the recognized solution of Maxwell's equations is discussed by the incoming of the dyadic Green's function. Our basic assumption is the inhomogeneous Maxwell's equations in free space which means an absence of dielectric or permeable media:

$$\nabla \times \mathbf{B} - \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \quad (1)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad (2)$$

It is frequently suitable to work with the Fourier transform and its inverse of an arbitrary function  $F$  which are defined as follows:

$$F(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} F(\mathbf{r}, t) e^{i\omega t} dt \quad (3)$$

$$F(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\mathbf{r}, \omega) e^{-i\omega t} d\omega \quad (4)$$

By combining the two Maxwell equations and eliminating the magnetic field, the second order equation for the electric field is obtained. A simple calculation establishes the wave equation in the following form,

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{E}(\mathbf{r}, t) - \nabla \nabla \cdot \mathbf{E}(\mathbf{r}, t) = \mu_0 \frac{\partial \mathbf{J}(\mathbf{r}, t)}{\partial t} \quad (5)$$

Applying Eq. (1) to the time dependent quantities in Eq. (5) gives wave equation in terms of the electric field in a complex form,

$$\left( \nabla^2 + \frac{\omega^2}{c^2} \right) \mathbf{E}(\mathbf{r}, \omega) - \nabla \nabla \cdot \mathbf{E}(\mathbf{r}, \omega) = -i\mu_0 \omega \mathbf{J}(\mathbf{r}, \omega) \quad (6)$$

Where  $\mu_0$  and  $c$  are the vacuum magnetic permeability and the speed of light respectively.

$\mathbf{E}(\mathbf{r}, t)$  and  $\mathbf{J}(\mathbf{r}, t)$  quantities are Fourier transformable. They can be substituted into wave equation to obtain

$$\mathcal{L} \mathbf{E}(\mathbf{r}, \omega) = -i\mu_0 \omega \mathbf{J}(\mathbf{r}, \omega) \quad (7)$$

Where  $\mathcal{L} = \nabla^2 + L_0^2 \vec{I} - \nabla \nabla$  is the linear operator relating  $\mathbf{J}$  and  $\mathbf{E}$  at a fixed frequency.  $\nabla$  is the derivative operator,  $\mathbf{r}$  is the radius vector,  $L_0$  is the scalar wave number,  $\vec{I}$  is the identity dyadic and  $\nabla \nabla$  is a double gradient which in a cartesian system of coordinates are defined by ,

$$\vec{I} = \sum_{m=1}^3 \sum_{n=1}^3 \mathbf{e}_m \mathbf{e}_n \delta_{mn}$$

$$\nabla\nabla = \sum_{m=1}^3 \sum_{n=1}^3 \mathbf{e}_m \mathbf{e}_n \frac{\partial}{\partial x_m} \frac{\partial}{\partial x_n}$$

Where  $x_i$  ( $i = 1, 2, 3$ ) are the Cartesian coordinates,  $\mathbf{e}_i$  ( $i = 1, 2, 3$ ) are the unit base vectors, and the symbol  $\delta_{mn}$  is the Kronecker delta, which is 1 for  $m = n$  and 0 for  $m \neq n$ .

The inverse linear operator  $\mathcal{L}^{-1}$  with the kernel  $\vec{\mathbf{G}}_\omega(\mathbf{r})$  can participate to calculate the field  $\mathbf{E}(\mathbf{r}, \omega)$  by the following relational equation

$$\mathbf{E}(\mathbf{r}, \omega) = -i\mu_0\omega\mathcal{L}^{-1}\mathbf{J}(\mathbf{r}, \omega) \quad (8)$$

The physical meaning of  $\vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0)$  is a field, measured at point  $\mathbf{r}$ , due to a unit point source  $\delta(\mathbf{r} - \mathbf{r}_0)$  localized at  $\mathbf{r}_0$ . Then it can be considered by

$$\vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0) = \mathcal{L}^{-1} \delta(\mathbf{r} - \mathbf{r}_0) \quad (9)$$

In free space an integral solution of the electric field  $\mathbf{E}(\mathbf{r}, \omega)$  is obtained from Eq.(9)

$$\mathbf{E}(\mathbf{r}, \omega) = -i\mu_0\omega \int \vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0) \cdot \mathbf{J}(\mathbf{r}_0, \omega) d^3\mathbf{r}_0 \quad (10)$$

$\vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0)$  is a  $3 \times 3$  tensor of second order. It is so called dyadic Green function. When the electric field is considered as a vector, the equivalent of the Green function is more complex. However, in the construction of  $\vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0)$ , in free space, is much simpler when the wave equation is used for the vector field  $\mathbf{E}(\mathbf{r}, \omega)$ . Moreover, it eliminates the selection of any fastidious gauge in the solution as Eq. (10) relates the source straightforwardly to the field. The solution of Eq. (9) in dyadic Green function is given by

$$\vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0) = -\left(\vec{\mathbf{I}} + \frac{\nabla\nabla}{L_0^2}\right) g_\omega(\mathbf{r} - \mathbf{r}_0) \quad (11)$$

In free space scalar Green function satisfies the Eq (A6). A spherical symmetric suitable solution can be accorded [11],

$$g_\omega(\mathbf{r} - \mathbf{r}_0) = \frac{e^{il_0 R}}{4\pi R} \quad (12)$$

Where  $R = |\mathbf{r} - \mathbf{r}_0|$ , the Eq(11) can be written in the form

$$\vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0) = -\left(\vec{\mathbf{I}} + \frac{\nabla\nabla}{L_0^2}\right) \frac{e^{il_0 R}}{4\pi R} \quad (13)$$

The solution of the electric field can be expressed in terms of the dyadic Green's function  $\vec{G}_\omega(\mathbf{r}, \mathbf{r}_0)$  as

$$\mathbf{E}(\mathbf{r}, \omega) = i\mu_0\omega \int \left( \vec{I} + \frac{\nabla\nabla}{L_0^2} \right) g_\omega(\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{J}(\mathbf{r}_0, \omega) d^3\mathbf{r}_0 \quad (14)$$

If the electrons distributive charge is in motion, it is localized in space much smaller than the wavelength. It is possible to develop, in space at a fixed time, the Dirac delta function in the Taylor series of the associated current density  $\mathbf{J}(\mathbf{r}_1, t)$  around the center of the distribution. It is located at  $\mathbf{r}_0$

$$\begin{aligned} \mathbf{J}(\mathbf{r}_1, t) = & \left( \int \mathbf{J}(\mathbf{r}', t) d^3\mathbf{r}' \right) \delta(\mathbf{r}_1 - \mathbf{r}_0) \\ & - \left( \int \mathbf{J}(\mathbf{r}', t) (\mathbf{r}' - \mathbf{r}_0) d^3\mathbf{r}' \right) \cdot \nabla \delta(\mathbf{r}_1 - \mathbf{r}_0) + \dots \end{aligned} \quad (15)$$

What is retained is the simplified first term in the series such that  $\mathbf{J}_0(t)\delta(\mathbf{r}_1 - \mathbf{r}_0)$ . In this stage it is convenient to express  $\mathbf{J}(\mathbf{r}_1, \omega)$  in terms of the Fourier transform of  $\mathbf{J}(\mathbf{r}_1, t)$ ,

$$\mathbf{J}(\mathbf{r}_1, \omega) = \int_{-\infty}^{\infty} \mathbf{J}(\mathbf{r}_1, t) e^{i\omega t} dt \quad (16)$$

By substituting the first term in integral of Eq. (16), the following equation is obtained

$$\mathbf{J}(\mathbf{r}_1, \omega) = \int_{-\infty}^{\infty} \mathbf{J}_0(t) \delta(\mathbf{r}_1 - \mathbf{r}_0) e^{i\omega t} dt \quad (17)$$

where  $\mathbf{J}_0(t)$  represents the Fourier transform inverse of  $\mathbf{J}_0(\omega_1)$ , the latter substituted in Eq(17) which can be fructuously written as,

$$\mathbf{J}(\mathbf{r}_1, \omega) = \frac{1}{2\pi} \iint_{-\infty}^{\infty} \mathbf{J}_0(\omega_1) \delta(\mathbf{r}_1 - \mathbf{r}_0(t)) e^{i(\omega - \omega_1)t} d\omega_1 dt \quad (18)$$

Where  $\omega$  and  $\omega_1$  are the radiative and oscillation frequencies respectively.

when Eq.(18) is substituted in Eq.(10) with regard to the Green's function propriety, it leads to the equation as follows,

$$\mathbf{E}(\mathbf{r}, \omega) = -\frac{i\mu_0\omega}{2\pi} \int_{-\infty}^{\infty} e^{i(\omega - \omega_1)t} dt \int \vec{G}_\omega(\mathbf{r}, \mathbf{r}_0(t)) \mathbf{J}_0(\omega_1) d\omega_1 \quad (19)$$

Where  $J_0(\omega_1)$  is the current density oscillates with a singularly principal frequency  $\omega_0$  and can be written in the form  $J_0\delta(\omega_1 - \omega_0)$ . The electric field is radiated at position vector  $\mathbf{r}_0(t)$  (in the source charge distribution), and an observer at position  $\mathbf{r}$  measures the field radiation at time  $t$ , the time delay for the field to travel from the source to the observer is  $\frac{|\mathbf{r}-\mathbf{r}_0(t')|}{c}$ , then the retarded time is  $t' = t - \frac{|\mathbf{r}-\mathbf{r}_0(t')|}{c}$ . Inserting Eq. (13) into Eq. (19) gives

$$\begin{aligned} \mathbf{E}(\mathbf{r}, \omega) &= -\frac{i\mu_0\omega}{2\pi} \int_{-\infty}^{\infty} e^{i(\omega-\omega_1)t} dt \int_{-\infty}^{\infty} \left( \mathbf{I} + \frac{\nabla\nabla}{L_0^2} \right) \frac{e^{il_0|\mathbf{r}-\mathbf{r}_0(t')|}}{4\pi|\mathbf{r}-\mathbf{r}_0(t')|} J_0(\omega_1) d\omega_1 \end{aligned} \quad (20)$$

The electric field in Eq. (20) is the total field from the source, including both radiative and non-radiative terms. In this integral, there is a dyadic form  $\left( \mathbf{I} + \frac{\nabla\nabla}{L_0^2} \right)$  applied at the scalar Green function  $g_\omega(\mathbf{r} - \mathbf{r}_0(t'))$  which can be simplified. It can indicate the only far radiative Green function related to  $\mathbf{r}_0(t')$  electron trajectory

$$\begin{aligned} \vec{\mathbf{G}}_\omega(\mathbf{r}, \mathbf{r}_0) &= -\frac{1}{4\pi} \left( \mathbf{I} - \frac{(\mathbf{r} - \mathbf{r}_0(t'))(\mathbf{r} - \mathbf{r}_0(t'))}{|\mathbf{r} - \mathbf{r}_0(t')|^2} \right) \frac{e^{il_0|\mathbf{r}-\mathbf{r}_0(t')|}}{|\mathbf{r} - \mathbf{r}_0(t')|} \\ &\quad + \mathcal{O}\left((\mathbf{r} - \mathbf{r}_0(t'))^{-2}\right) \end{aligned} \quad (21)$$

The Terms which are of the following superior order  $|\mathbf{r} - \mathbf{r}_0(t')|^{-1}$  are ignored. They do not contributed to the radiation field. The unit vector in this expression  $\frac{\mathbf{r}-\mathbf{r}_0(t')}{|\mathbf{r}-\mathbf{r}_0(t')|} = \mathbf{n}$ , is the direction from the retarded position of the electron to the observer as is located( $\mathbf{r}$ ). Under this statement the observer is situated in the far-zone which falls off as  $\frac{1}{|\mathbf{r}-\mathbf{r}_0|}$ . As a consequence, in accordance with [11], it is rewritten as  $\frac{e^{il_0|\mathbf{r}-\mathbf{r}_0(t')|}}{|\mathbf{r}-\mathbf{r}_0(t')|} \approx \frac{e^{il_0|\mathbf{r}|}}{|\mathbf{r}|}$ . Changing the variable from the observation time  $t$  to the emission time  $t'$  after substituting Eq. (21) into Eq. (20) the field in dyadic notation is obtained accordingly as,

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\mu_0\omega}{8\pi^2} \frac{e^{il_0(|\mathbf{r}|)}}{|\mathbf{r}|} \int_{-\infty}^{\infty} \left( \mathbf{I} - \mathbf{n}(t')\mathbf{n}(t') \right) J_0 e^{i[(\omega-\omega_0)(t' + \frac{|\mathbf{r}-\mathbf{r}_0(t')|}{c})]} (1 - \mathbf{n}(t') \cdot \boldsymbol{\beta}(t')) dt' \quad (22)$$

While the study point is assumed to be distant from the area of space, the unit vector  $\mathbf{n}$  is nearly constant in time. Moreover the distance  $|\mathbf{r} - \mathbf{r}_0(t')|$  is almost written by  $|\mathbf{r} - \mathbf{r}_0(t')| \approx |\mathbf{r}| - \mathbf{n}(t') \cdot \mathbf{r}_0(t')$ .

According to the electromagnetic theory, dyadic details are often used succinctly. If the reader becomes customary with the notations, it can be used in some circumstances once manipulated by vector operations. A simple example given in current work is the operator  $(\vec{I} - \mathbf{n}\mathbf{n})$  which is applied to  $\mathbf{J}_0$ . It gathers the componential current transversely directed to the furthest sight. The latter operator which appears in the electric field integrals can be represented by  $\mathbf{J}_0 - (\mathbf{n} \cdot \mathbf{J}_0)\mathbf{n}$ . Then Eq. (22) can be written as

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\mu_0\omega}{8\pi^2} \frac{e^{iI_0(|\mathbf{r}|)}}{|\mathbf{r}|} \int_{-\infty}^{\infty} [\mathbf{J}_0 - (\mathbf{n} \cdot \mathbf{J}_0)\mathbf{n}] e^{i\left[(\omega - \omega_0)\left(t' + \frac{|\mathbf{r} - \mathbf{r}_0(t')|}{c}\right)\right]} (1 - \mathbf{n}(t') \cdot \boldsymbol{\beta}(t')) dt' \quad (23)$$

### 3. REGULAR AND ANOMALOUS DOPPLER EFFECT

Obviously, the integral in time of Eq. (23) accounts for the frequency  $\omega$  of the radiative field which is not the same as the oscillation frequency  $\omega_0$  of the source. This difference is caused by the motion of the source characterized by its velocity  $\mathbf{v}_0(t')$ . If the argument of the exponential term of the latter expression is developed in the far field region of the source the following equation is attained,

$$(\omega - \omega_0)\left(t' + \frac{|\mathbf{r} - \mathbf{r}_0(t')|}{c}\right) = (\omega - \omega_0)\left(t' + \frac{|\mathbf{r}|}{c}\right) - \int (\omega - \omega_0) \mathbf{n}(t') \cdot \frac{\mathbf{v}_0(t')}{c} dt' \quad (24)$$

The phase of the integral form of the electric field shows that it is affected by the source velocity. This expression shows that any motion of the source makes the radiative Green function be related to  $\mathbf{r}_0(t')$  in order to take an oscillation frequency  $(\omega - \omega_0) \mathbf{n}(t') \cdot \frac{\mathbf{v}_0(t')}{c}$  which surely yields the electric field frequency. It is accountable for the Doppler effect inducing a retarded time. Defining a condition for the far-field zone by assuming that  $\mathbf{n}$  does not vary with time ( $\frac{d\mathbf{n}}{dt} = 0$ ) the Eq. (24) can be written this way,

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\mu_0\omega}{8\pi^2} \frac{e^{iI_0(|\mathbf{r}|)}}{|\mathbf{r}|} [\mathbf{J}_0 - (\mathbf{n} \cdot \mathbf{J}_0)\mathbf{n}] \int_{-\infty}^{\infty} e^{i\left[(\omega - \omega_0)\left(t' + \frac{|\mathbf{r}|}{c}\right) - \int (\omega - \omega_0) \mathbf{n} \cdot \frac{\mathbf{v}_0(t')}{c} dt'\right]} (1 - \mathbf{n} \cdot \boldsymbol{\beta}(t')) dt' \quad (25)$$

Any type of the source trajectory can be applied. For example an electric point source characterized by uniform motion can be chosen. The Fourier transform of the electric field in this case is

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\mu_0\omega}{4\pi} \frac{e^{iI_0(|\mathbf{r}|)}}{|\mathbf{r}|} [\mathbf{J}_0 - (\mathbf{n} \cdot \mathbf{J}_0)\mathbf{n}] \left(1 - \mathbf{n} \cdot \frac{\mathbf{v}_0}{c}\right) \delta\left(\omega \left(1 - \mathbf{n} \cdot \frac{\mathbf{v}_0}{c}\right) - \omega_0\right) \quad (26)$$

The Dirac delta function exit only if the frequency takes the following value

$$\omega = \frac{\omega_0}{\left(1 - \mathbf{n} \cdot \frac{\mathbf{v}_0}{c}\right)}$$

which is the celebrated relation for the usual or ordinary Doppler effect. Besides, in the far region, the local of the constant phase is considered as spherical surfaces, approximated by the plane ones which are equidistant.

In the near zone,  $l_0 |\mathbf{r} - \mathbf{r}_0(t')| \ll 1$  is necessarily attained. The wavelength is much bigger than the space where the electric field is relatively calculated in proportion to the source position. The radiative Green function in the near zone becomes

$$\vec{\mathcal{G}}_\omega(\mathbf{r}, \mathbf{r}_0) = \frac{1}{4\pi l_0^2} \left( \vec{\mathcal{I}} - \frac{3(\mathbf{r} - \mathbf{r}_0(t'))(\mathbf{r} - \mathbf{r}_0(t'))}{|\mathbf{r} - \mathbf{r}_0(t')|^2} \right) \frac{e^{il_0|\mathbf{r} - \mathbf{r}_0(t')|}}{|\mathbf{r} - \mathbf{r}_0(t')|^3} \quad (27)$$

If the small distance dependent phase shift is contained in the  $e^{il_0|\mathbf{r} - \mathbf{r}_0(t')|}$  factor, it can be neglected. It will be discussed later. Then the near zone Green function is perhaps described as a quasi-static Green function. It crops up as the following,

$$\vec{\mathcal{G}}_\omega(\mathbf{r}, \mathbf{r}_0) = \frac{1}{4\pi l_0^2} \frac{(\vec{\mathcal{I}} - 3\mathbf{nn})}{|\mathbf{r} - \mathbf{r}_0(t')|^3} \quad (28)$$

In the near-field interactions based on Eq.(28), inducing retardation effects are neglected Eq. (20) becomes

$$\mathbf{E}(\mathbf{r}, \omega) = -\frac{i\mu_0\omega}{2\pi} \int_{-\infty}^{\infty} e^{i(\omega - \omega_1)t} dt \int_{-\infty}^{\infty} \frac{1}{4\pi l_0^2} \frac{(\vec{\mathcal{I}} + 3\mathbf{nn})}{|\mathbf{r} - \mathbf{r}_0|^3} J_0 \delta(\omega_1 - \omega_0) d\omega_1 \quad (29)$$

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{i\mu_0\omega}{2\pi} \frac{1}{4\pi l_0^2} \int_{\Delta t} e^{i(\omega - \omega_0)t} \frac{(\vec{\mathcal{I}} - 3\mathbf{nn})}{|\mathbf{r} - \mathbf{r}_0|^3} J_0 dt \quad (29)$$

where  $\Delta t$  is the set times during which the integration is accomplished. While  $\Delta t$  has to gratify the obligation  $|\mathbf{r} - \mathbf{r}_0 t| \ll \frac{\lambda}{2\pi}$ , its duration does not possess a definite significant value, namely that the integral in Eq. (30) which is not adequately defined. In addition, the wave number  $l_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c} \rightarrow 0$  so that the delay effects become negligible and the phase is imprecise. It is essentially responsible for the irregular characteristics of the near-region Doppler effect. Under these conditions, we are in the limit of the electrostatic approximation. In other terms, the spectrum of the radiation field in the near zone is ambiguous. The field which varies as  $\frac{1}{|\mathbf{r} - \mathbf{r}_0|^3}$  is the electrostatic field. This field is dominant in the close vicinity of the source while its amplitude diminishes quickly as a function of distance. Actually, this finding is construed as follows: taking account of the source motion, the immobile observer can in no way stay sometime in the near field zone to obtain all the information on the field spectrum. Similarly, the spectrum of the radiative field in the middle zone, where  $|\mathbf{r} - \mathbf{r}_0 t| \approx \frac{\lambda}{2\pi}$ , can never be achieved. In this respect, the variation of the dominant field is as  $\frac{1}{|\mathbf{r} - \mathbf{r}_0|^2}$ . This field is prolonged a little beyond the electrostatic field. Nevertheless, it decreases rapidly



with distance. Thus, although the radiation electric field exists in the regions near and middle field, their spectrum cannot be obtained in both regions. Hence, the Doppler effect is justified restrictively in the far-field area.

#### 4. CONCLUSION

The fields may be split into three kinds depending on their variation in distance. Briefly, in the vicinity of the source, it is an electrostatic field. In fact, the wavelength is much bigger than the space where the electric field is relatively calculated in proportion to the source position. A little beyond the latter, is the induction field. The critical region affects the propagation of electromagnetic field. In fact, both fields combined together in an EM wave are needed to transport energy from one position to another. But, induction fields can, pass no energy as they consist of magnetic or electric field only. Once it is defined far away from the source, it is the radiated field. Solely, in the far field the Fourier transform exists. It is the origin of the existence of the phase of a plane wave. Principally, the Doppler effect is a result of the source motion and explains the condition of its existence.

When the observer is in the source's far-zone field, the Doppler effect is entirely regular. It signifies that the Doppler shift is identical to all the field components and it is independent of space with the source as a point of departure. When the observer is in the near-zone field the Doppler effect is anomalous.

In the further research, it is possible to concentrate on the differences rather than the similarities between the emission of the electric field in the free and the undulated bounded space.

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